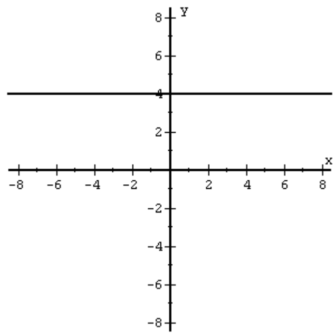
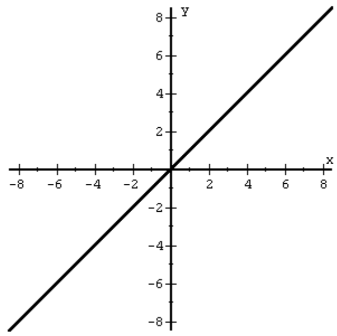
PARENT FUNCTIONS



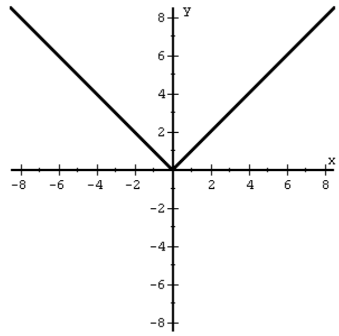
$f(x) = a$

Constant



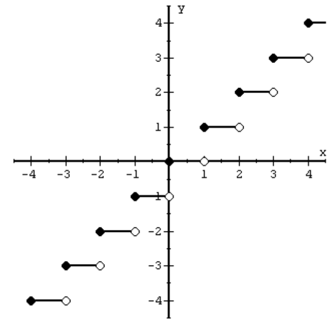
$f(x) = x$

Linear



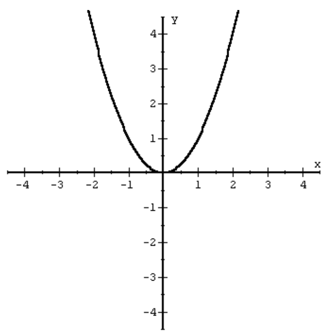
$f(x) = |x|$

Absolute Value



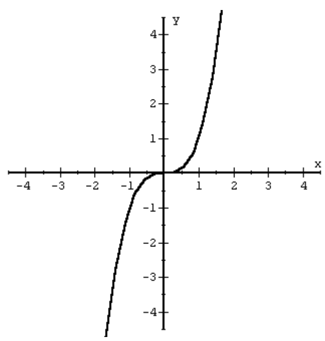
$f(x) = \text{int}(x) = [x]$

Greatest Integer



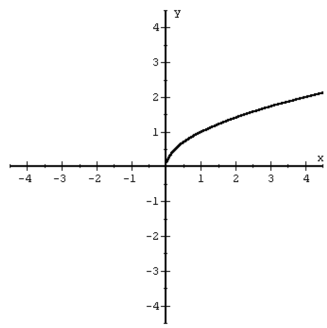
$f(x) = x^2$

Quadratic



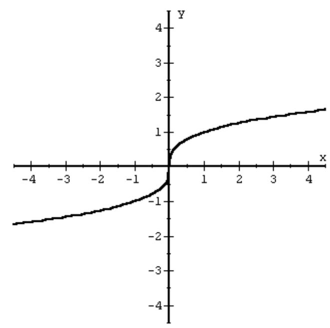
$f(x) = x^3$

Cubic



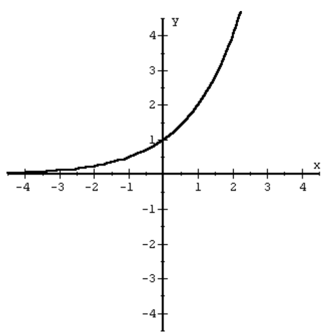
$f(x) = \sqrt{x}$

Square Root



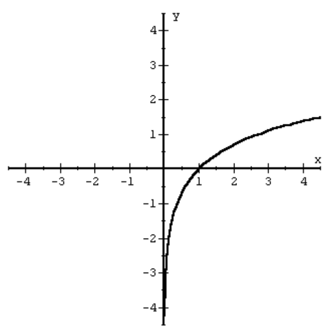
$f(x) = \sqrt[3]{x}$

Cube Root



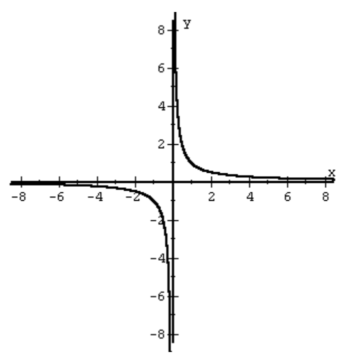
$f(x) = a^x$

Exponential



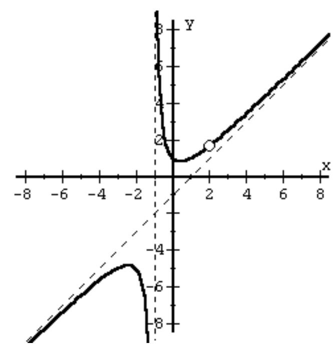
$f(x) = \log_a x$

Logarithmic



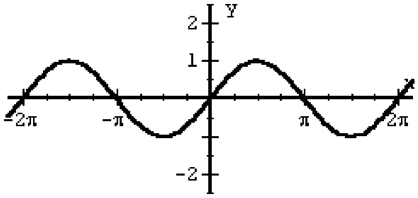
$f(x) = \dfrac{1}{x}$

Reciprocal

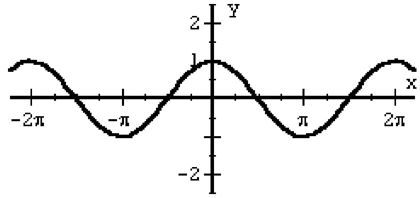


$f(x) = \dfrac{(x^2+1)(x-2)}{(x+1)(x-2)}$

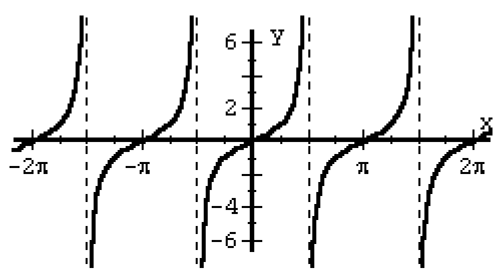
Rational



$f(x) = \sin x$



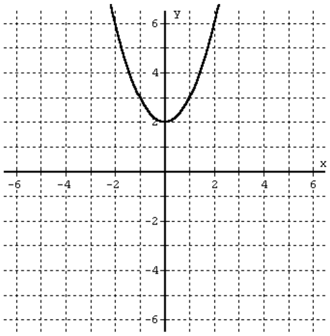
$f(x) = \cos x$



$f(x) = \tan x$

Trigonometric Functions

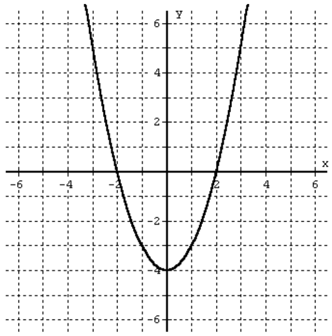
Transformations of $y = f(x) = x^2$



Vertical Shift – Up 2

$y = x^2 + 2$

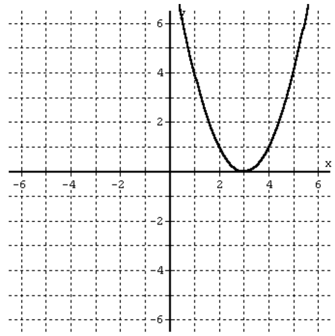
$y = f(x) + 2$



Vertical Shift – Down 4

$y = x^2 - 4$

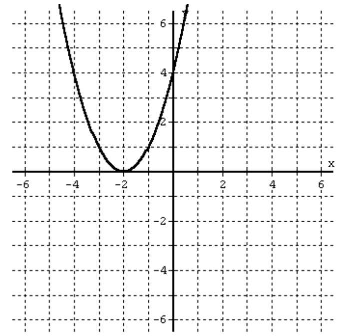
$y = f(x) - 4$



Horizontal Shift – Right 3

$y = (x - 3)^2$

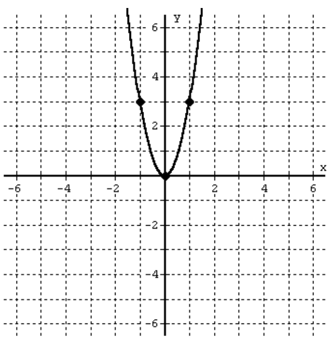
$y = f(x - 3)$



Horizontal Shift – Left 2

$y = (x + 2)^2$

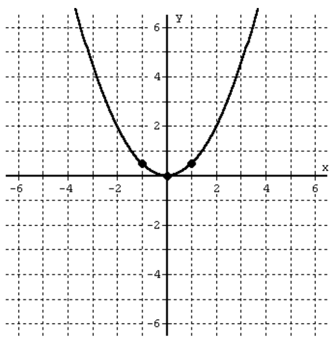
$y = f(x + 2)$



Vertical Stretch
Steeper / Narrower

$y = 3x^2$

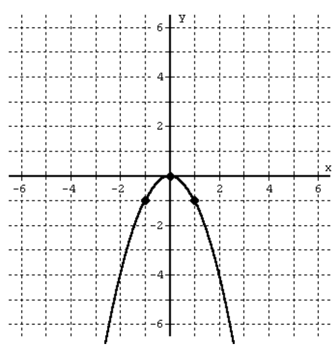
$y = 3f(x)$



Vertical Compression
Flatter / Wider

$y = \frac{1}{2}x^2$

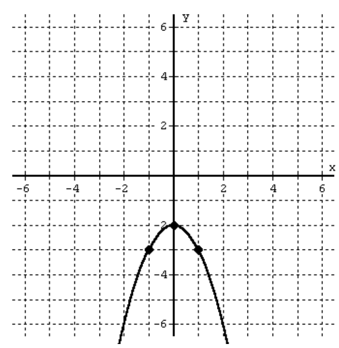
$y = \frac{1}{2}f(x)$



Reflection about the x-axis
Upside Down

$y = -x^2$

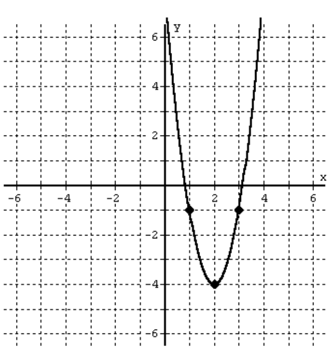
$y = -f(x)$



Upside Down
and Down 2

$y = -x^2 - 2$

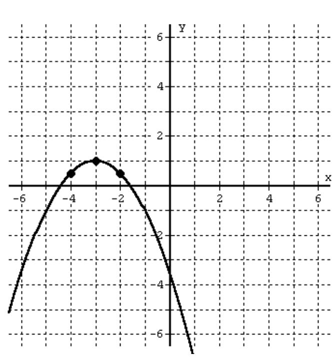
$y = -f(x) - 2$



Steeper, Opens Up, Right 2, Down 4

$y = 3(x - 2)^2 - 4$

$y = 3f(x - 2) - 4$



Wider, Opens Down, Left 3, Up 1

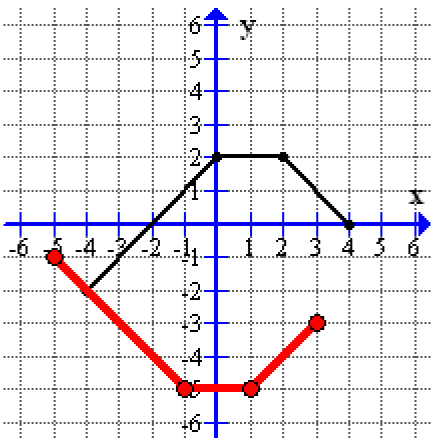
$y = -\frac{1}{2}(x + 3)^2 + 1$

$y = -\frac{1}{2}f(x + 3) + 1$

Graphing Transformations Worksheet 5

A piecewise-defined function $f(x)$ is graphed in black. Another function $g(x)$, which is a transformation of $f(x)$, is graphed in red (thicker graph). Find an algebraic expression for the transformed function $g(x)$ in terms of $f(x)$. In general, $g(x) = \pm a\, f\left(\pm b\,(x - c)\right) + d$

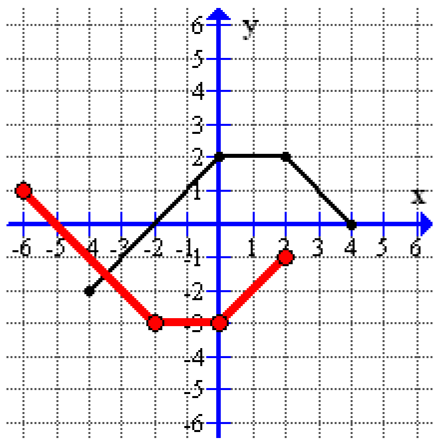
Hint:
1. Pick a point on $f(x)$ and locate the corresponding point on $g(x)$
2. Perform Reflection(s): about the x-axis and/or about the y-axis
3. Perform Compression(s)/Expansion(s): horizontal and/or vertical
4. Perform Translation(s): up/down and/or left/right
5. Test your function with several points on $g(x)$



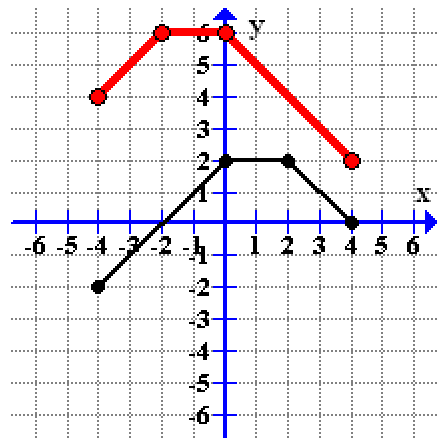
ex) $g(x) = -f(x+1) - 3$

test $g(-5) = -f(-5+1) - 3 = -1$

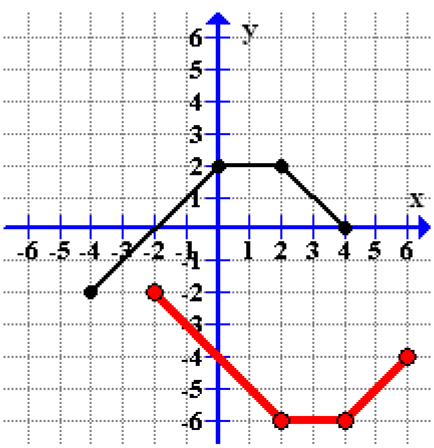
test $g(2) = -f(2+1) - 3 = -4$



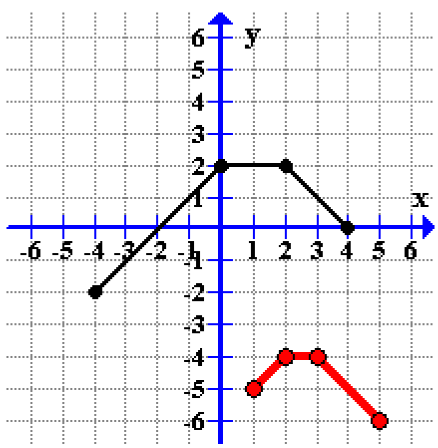
2) $g(x) =$



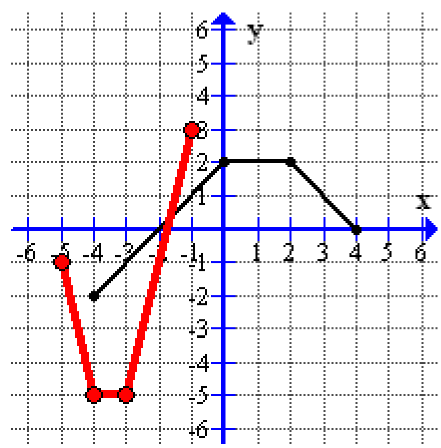
3) $g(x) =$



4) $g(x) =$



5) $g(x) =$



6) $g(x) =$

Continuous Functions - Definitions

<u>Continuity (Informal Definition)</u>

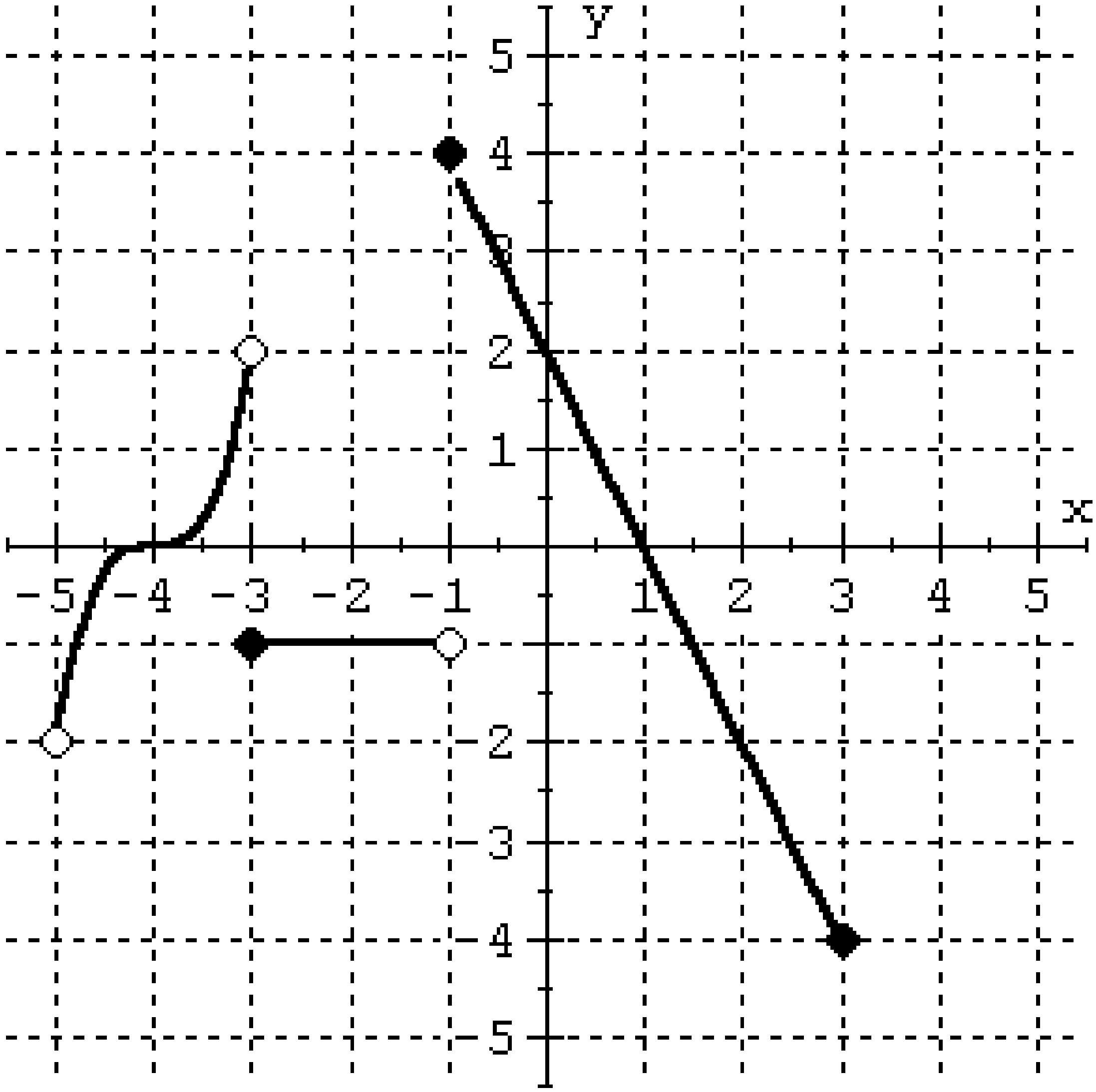
A function is **continuous** over an interval of its domain if its hand-drawn graph over that interval can be sketched without lifting the pencil from the paper.

<u>Continuity (Limit Definition)</u>

A function f is continuous at $x = a$ if and only if $\lim_{x \to a} f(x) = f(a)$;
that is, $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$.

<u>Continuity: (ε , δ Definition)</u>

A function f is continuous at $x = a$ if and only if
$\forall e > 0, \exists d > 0,$ such that
$\left| f(x) - f(a) \right| < e$ if $\left| x - a \right| < d$.



$\lim_{x \to -3^-} f(x) =$

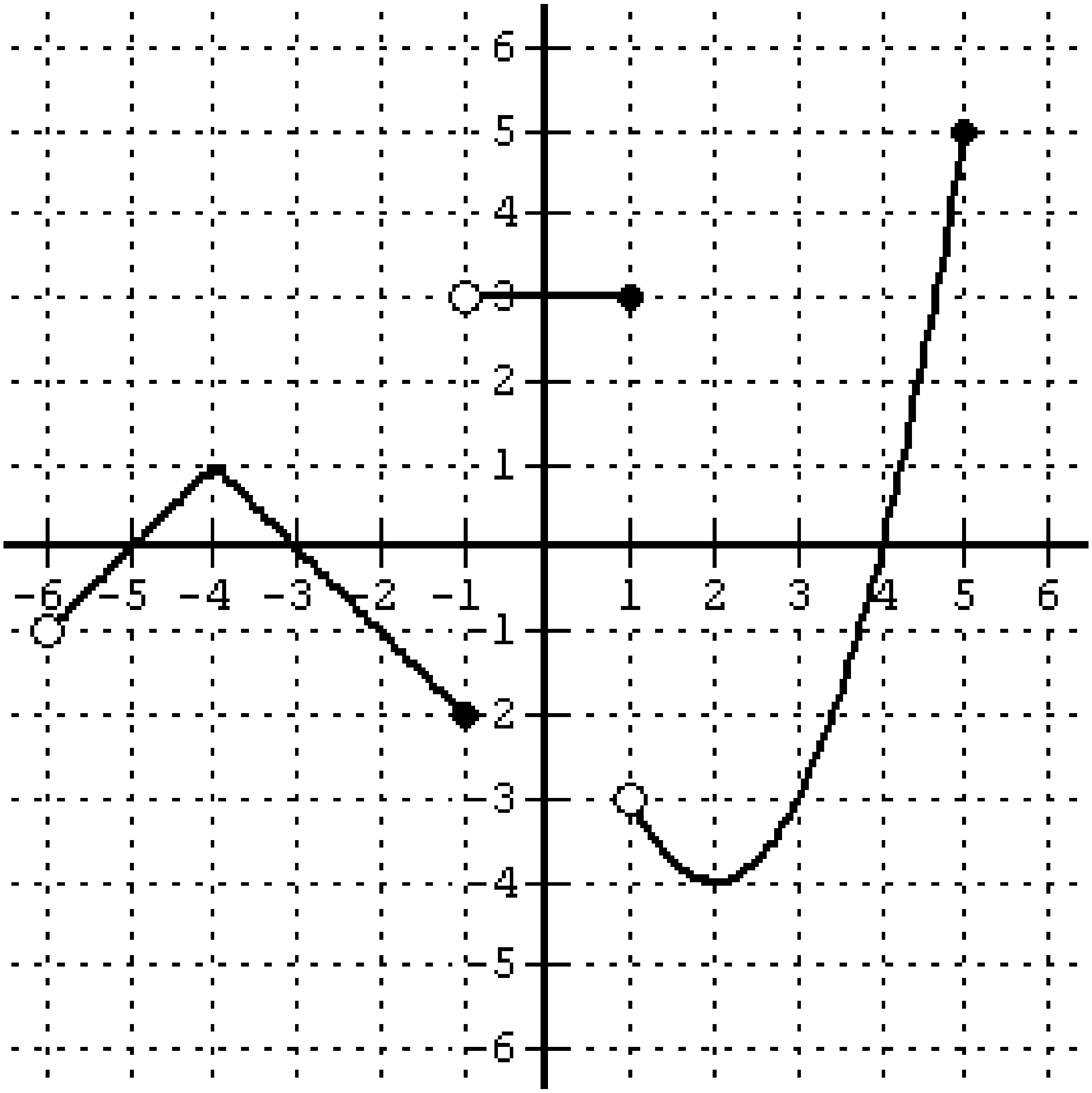
$\lim_{x \to -1^-} f(x) =$

$\lim_{x \to -3^+} f(x) =$

$\lim_{x \to -1^+} f(x) =$

$\lim_{x \to -3} f(x) =$

$\lim_{x \to -1} f(x) =$



$\lim_{x \to -1^-} f(x) =$

$\lim_{x \to 1^-} f(x) =$

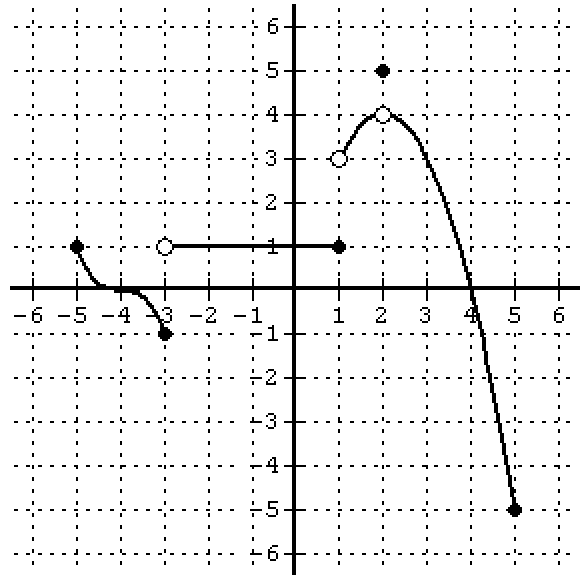
$\lim_{x \to -1^+} f(x) =$

$\lim_{x \to 1^+} f(x) =$

$\lim_{x \to -1} f(x) =$

$\lim_{x \to 1} f(x) =$

1)A) $\lim_{x \to -3^-} f(x)$



B) $\lim_{x \to -3^+} f(x)$

C) $\lim_{x \to -3} f(x)$

D) $\lim_{x \to 1^-} f(x)$

E) $\lim_{x \to 1^+} f(x)$

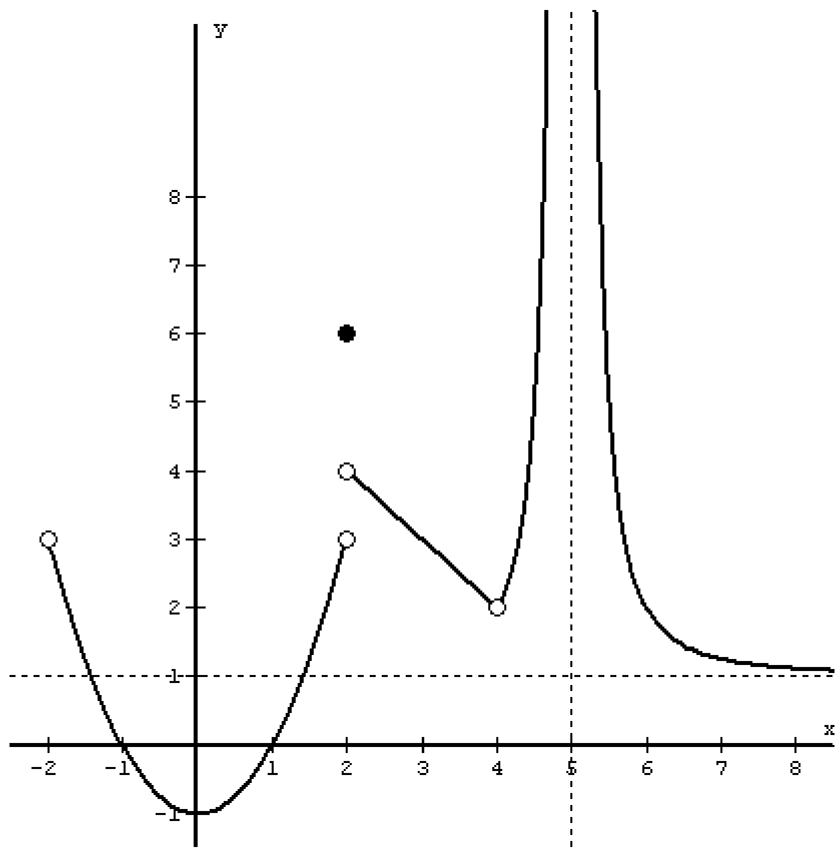
F) $\lim_{x \to 1} f(x)$

G) $\lim_{x \to 2} f(x)$

H) $f(-3)$ I) $f(1)$ J) $f(2)$

---

2)A) $\lim_{x \to -2^+} f(x) =$



B) $f(2) =$

C) $\lim_{x \to 4} f(x) =$

D) $\lim_{x \to 2} f(x) =$

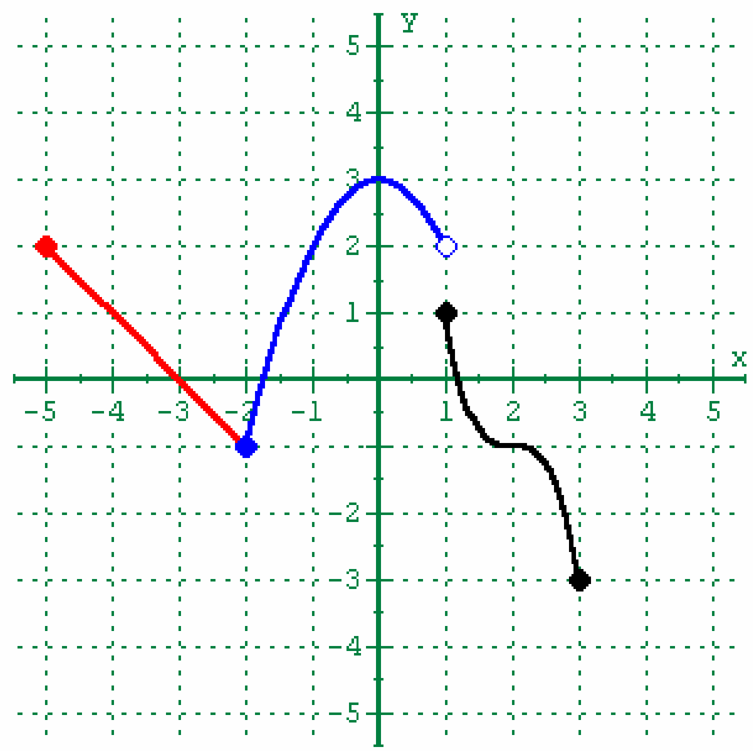
E) $\lim_{x \to 5} f(x) =$

F) $f(4) =$

G) $\lim_{x \to 2^-} f(x) =$

H) $\lim_{x \to \infty} f(x) =$

Piecewise-Defined Functions 2



$$f(x) = \begin{cases} & \text{for} \\ & \text{for} \\ & \text{for} \end{cases}$$

Find $f(-4)$

$f(-2)$

$f(0)$

$f(1)$

Find the x-intercept(s)

Find the y-intercept(s)

Domain:

Range:

Maximum:

Minimum

Find the Interval(s) on x where the function is:

Increasing:

Decreasing:

Constant:

Combinations of Functions

Let $f(x) = x^2 - 4$ and $g(x) = 3x + 1$

Sum:

$$f(x) + g(x) = (f + g)(x)$$
$$\left(x^2 - 4\right) + \left(3x + 1\right) = x^2 + 3x - 3$$

Difference:

$$f(x) - g(x) = (f - g)(x)$$
$$\left(x^2 - 4\right) - \left(3x + 1\right) = x^2 - 3x - 5$$

Product:

$$f(x) \cdot g(x) = (fg)(x)$$
$$\left(x^2 - 4\right) \cdot \left(3x + 1\right) = 3x^3 + x^2 - 12x - 4$$

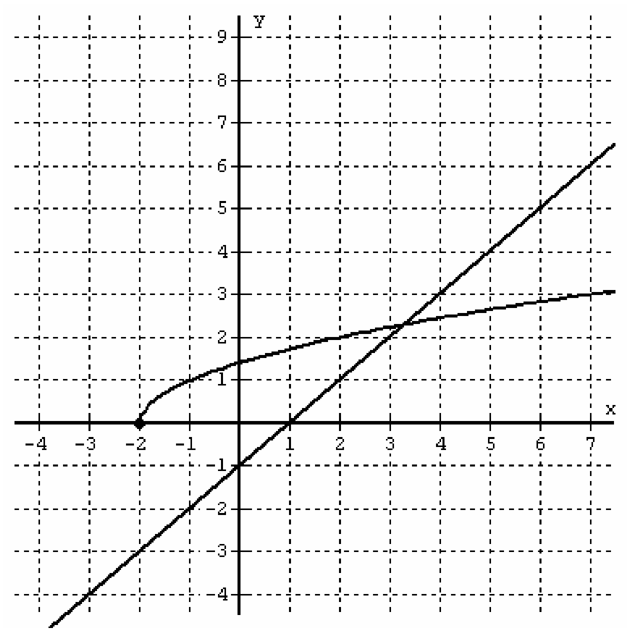
Quotient:

$$\frac{f(x)}{g(x)} = \frac{x^2 - 4}{3x + 1} = \left(\frac{f}{g}\right)(x) \quad \text{for } g(x) \neq 0$$

The Domain of the Combination of two functions is the Intersection of the Domains of those two functions.

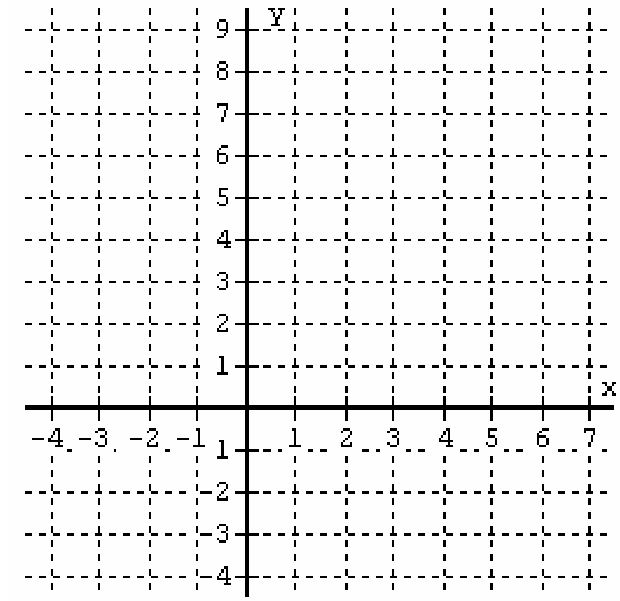
$$\text{Domain}_{f \pm g} = \text{Domain}_f \cap \text{Domain}_g$$
$$\text{Domain}_{f \cdot g} = \text{Domain}_f \cap \text{Domain}_g$$
$$\text{Domain}_{f/g} = \text{Domain}_f \cap \text{Domain}_g \cap \{x \mid g(x) \neq 0\}$$

Addition of Ordinates



$f(x) = \sqrt{x+2}$

$g(x) = x - 1$

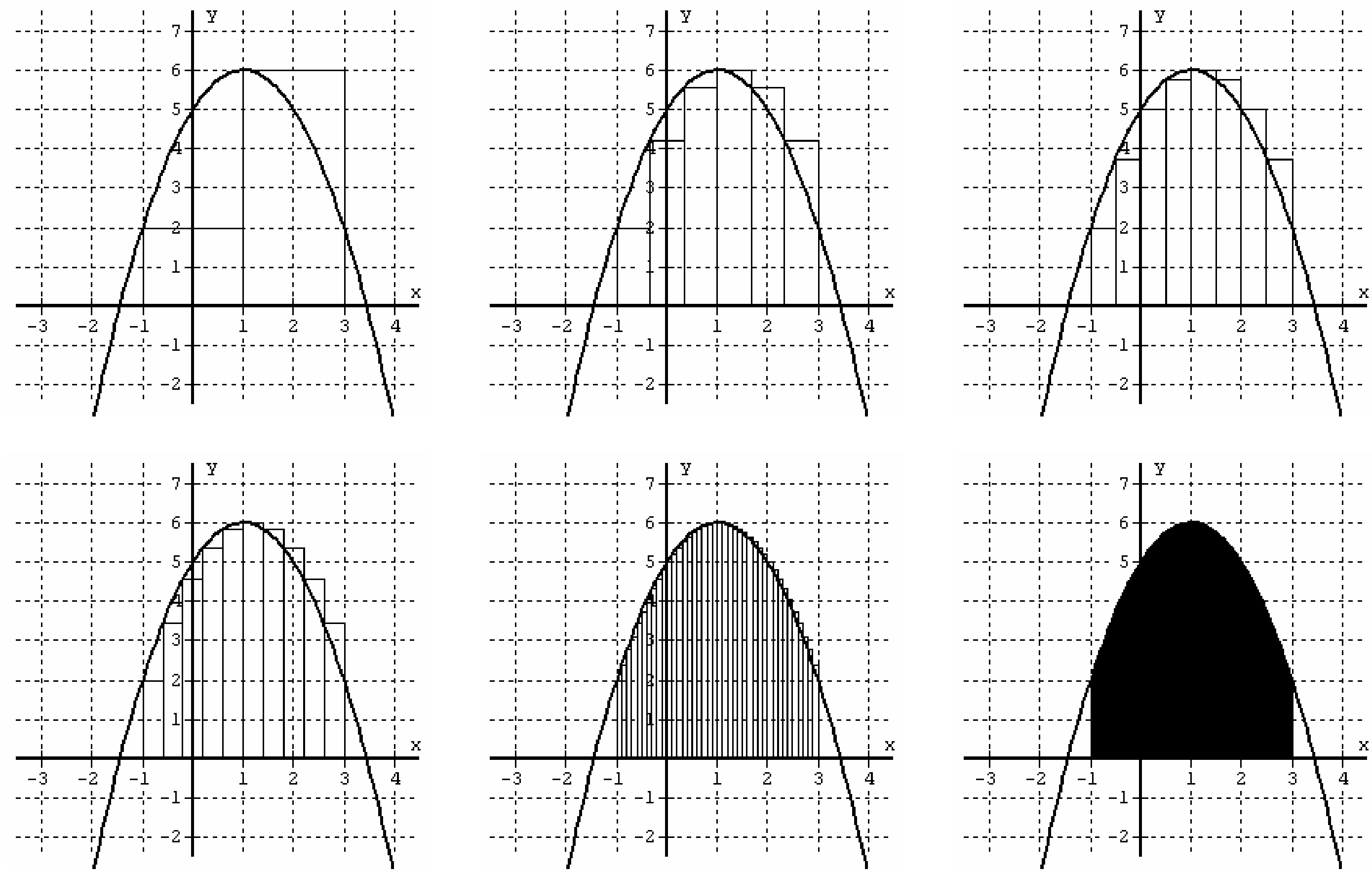


$y = (f + g)(x)$

Graph $y = (f+g)(x) = \sqrt{x+2} + x - 1$

x	$f(x) = \sqrt{x+2}$	$g(x) = x - 1$	$f(x) + g(x)$
– 3	undefined	– 4	undefined
– 2	0	– 3	– 3
– 1	1	– 2	– 1
0	$\sqrt{2} \approx 1.41$	– 1	0.41
1	$\sqrt{3} \approx 1.73$	0	1.73
2	2	1	3
3	$\sqrt{5} \approx 2.24$	2	4.24
4	$\sqrt{6} \approx 2.45$	3	5.45
5	$\sqrt{7} \approx 2.65$	4	6.65
6	$\sqrt{8} \approx 2.83$	5	7.83
7	3	6	9

Estimating the Area Under a Curve - Limits



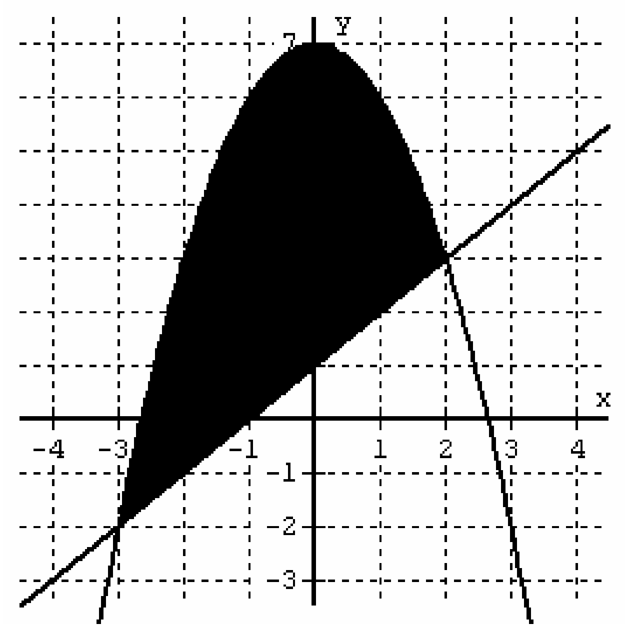
Number of Rectangles	Estimated Area: Riemann Sum
2	16
4	18
6	18.37037
8	18.5
10	18.56
20	18.64
40	18.66
100	18.6656
500	18.66662
1000	18.66666

The Limit of the Estimated Area, using Riemann Sums, is $18\frac{2}{3}$

Using the Definite Integral in Calculus,

$$\text{Area} = \int_{-1}^{3} -(x-1)^2 + 6 \; dx = 18\frac{2}{3}$$

Area Between Two Curves



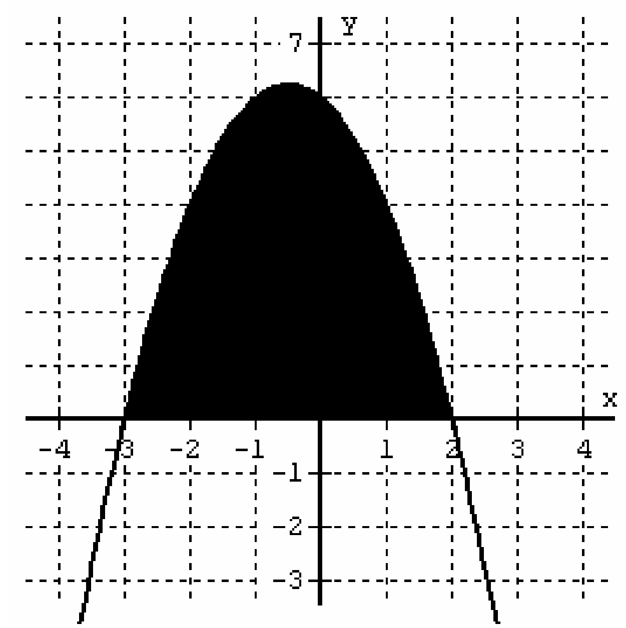
$$f(x) = -x^2 + 7$$
$$g(x) = x + 1$$

$$\text{Area} = \int_a^b f(x) - g(x)\, dx$$

$$\text{Area} = \int_{-3}^{2} \left(-x^2 + 7\right) - \left(x + 1\right) dx$$

$$\text{Area} = 20\frac{5}{6}$$

Area Under the Difference Function



$$f(x) = -x^2 + 7$$
$$g(x) = x + 1$$
$$\left(f - g\right)(x) = -x^2 - x + 6$$

$$\text{Area} = \int_a^b \left(f - g\right)(x)\, dx$$

$$\text{Area} = \int_{-3}^{2} -x^2 - x + 6\, dx$$

$$\text{Area} = 20\frac{5}{6}$$

Repeated Zeros of a Polynomial

<u>Example 1</u>

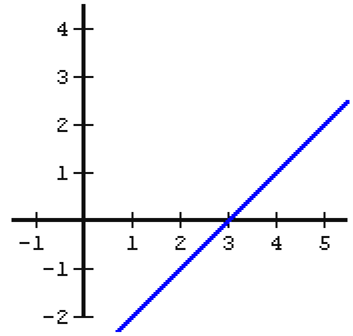
$f(x) = (x-3)^1 = 0$

$x - 3 = 0$

$x = 3$

$x = 3$ is a zero of multiplicity 1

We say $x = 3$ is a non-repeated zero



$f(x) = (x-3)^1$

Note: A (Real) **zero of multiplicity 1** : Crosses the x-axis rather steeply like a line.

Definition: If the factor $(x - r)$ occurs more than once in the factorization of f, then r is called a **repeated zero**, or **multiple zero** of f.

<u>Example 2</u>

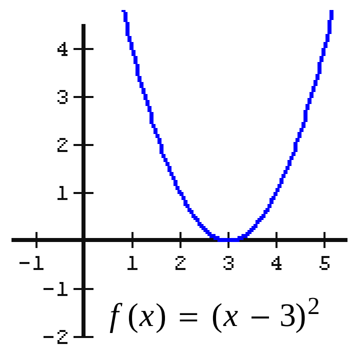
$f(x) = x^2 - 6x + 9 = 0$

$(x-3)^2 = 0$

$(x-3)(x-3) = 0$

$x = 3$ or $x = 3$

$x = 3$ is a zero of multiplicity 2



Note: A (Real) **zero of multiplicity 2** : Just touches the x-axis but does not cross the x-axis

Gets "flat" near the x-axis

"Looks" like $y = \pm x^2$ at the x-intercept

<u>Example 3</u>

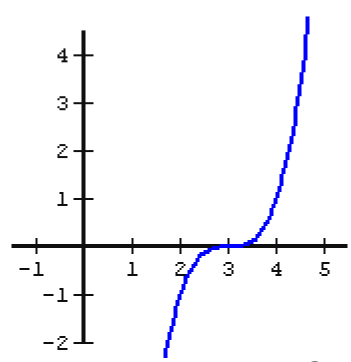
$f(x) = x^3 - 9x^2 + 27x - 27 = 0$

$(x-3)^3 = 0$

$(x-3)(x-3)(x-3) = 0$

$x = 3$ or $x = 3$ or $x = 3$

$x = 3$ is a zero of multiplicity 3



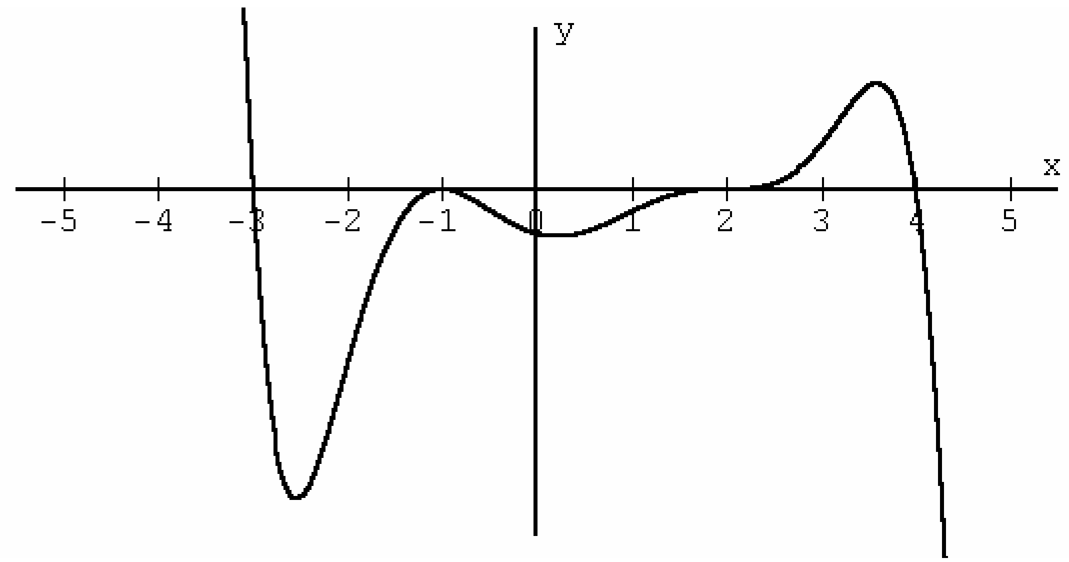
$f(x) = (x-3)^3$

Note: A (Real) **zero of multiplicity 3** : Crosses the x-axis

Gets "flat" near the x-axis

"Looks" like $y = \pm x^3$ at the x-intercept

FACTORING A POLYNOMIAL USING ITS GRAPH



X-intercepts: $x = -3$ $\quad x = -1$ $\quad x = 2$ $\quad x = 4$

$(x + 3) = 0 \quad (x + 1) = 0 \quad (x - 2) = 0 \quad (x - 4) = 0$

Multiplicity of Zeros: 1 $\quad$ 2 $\quad$ 3 $\quad$ 1

Factors: $(x + 3)^1 \quad (x + 1)^2 \quad (x - 2)^3 \quad (x - 4)^1$

Polynomial: $f(x) = -(x + 3)(x + 1)^2(x - 2)^3(x - 4)$

$f(x) = -x^7 + 5x^6 + 7x^5 - 57x^4 + 26x^3 + 124x^2 - 56x - 96$

<u>Facts:</u>

$f(x) = a_n x^n + \ldots + a_1 x + a_0$

A polynomial of degree **n** can have :
A) At Most **n** zeros
B) At Most **n** factors
C) At Most **n** x-intercepts
D) At Most **(n – 1)** turning points (humps)

A polynomial of degree 7 can have :
A) At Most 7 zeros
B) At Most 7 factors
C) At Most 7 x-intercepts
D) At Most 6 turning points (humps)

ASYMPTOTES

I Vertical Asymptotes

A line $x = a$ is called a **vertical asymptote** of the graph of a function f if $f(x) \to \infty$ or $f(x) \to -\infty$ as x approaches a from the left or right.

$$\lim_{x \to a} \left| f(x) \right| = \infty \quad \Leftrightarrow \quad x = a \text{ is a vertical asymptote}$$

II Horizontal Asymptotes

A line $y = L$ is called a **horizontal asymptote** of the graph of a function f if $f(x) \to L$ as x approaches ∞ or as x approaches $-\infty$.

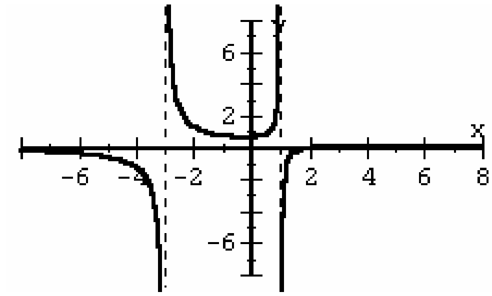
$$\left. \begin{array}{c} \lim\limits_{x \to +\infty} f(x) = L \\ \text{or} \\ \lim\limits_{x \to -\infty} f(x) = L \end{array} \right\} \quad \Leftrightarrow \quad y = L \text{ is a horizontal asymptote}$$

Horizontal and Slant Asymptotes

Case 1

If deg(Denominator) > deg(Numerator), then the Horizontal Asymptote is $y = 0$.

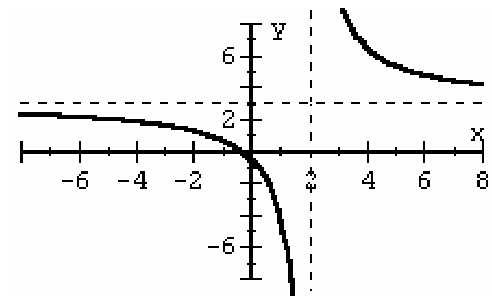
ex) $f(x) = \dfrac{x-2}{x^2+2x-3} = \dfrac{x-2}{(x-1)(x+3)}$



Case 2

If deg(Numerator) = deg(Denominator), then the Horizontal Asymptote is $y = \dfrac{a_n}{b_n}$ where $y = \dfrac{a_n}{b_n}$ is the ratio of the leading coefficients.

ex) $f(x) = \dfrac{3x+1}{x-2} = \dfrac{\boxed{3}\,x+1}{\boxed{1}\,x-2}$



Horizontal asymptote is $y = \dfrac{3}{1} = 3$

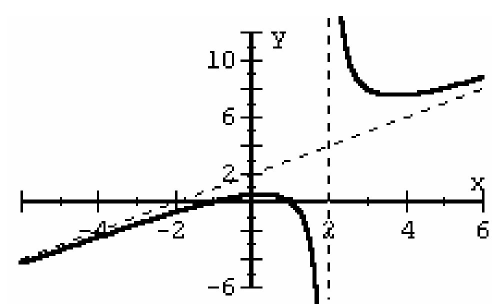
Case 3

If deg(Numerator) = deg(Denominator) + 1, then the graph has a Slant Asymptote.
An equation of the Slant Asymptote is $y = mx + b$, where m and b may be determined by long division.

ex) $f(x) = \dfrac{x^2-1}{x-2} = \boldsymbol{x+2} + \dfrac{3}{x-2}$

$$\begin{array}{r} x+2 \\ x-2\,\overline{)\,x^2+0x-1} \\ \underline{-(x^2-2x)} \\ 2x-1 \\ \underline{-(2x-4)} \\ 3 \end{array}$$

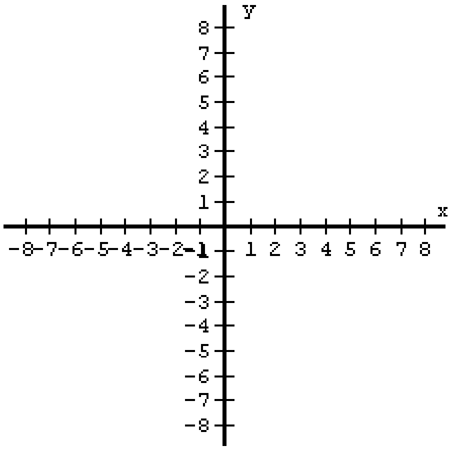
$\leftarrow$ slant asymptote

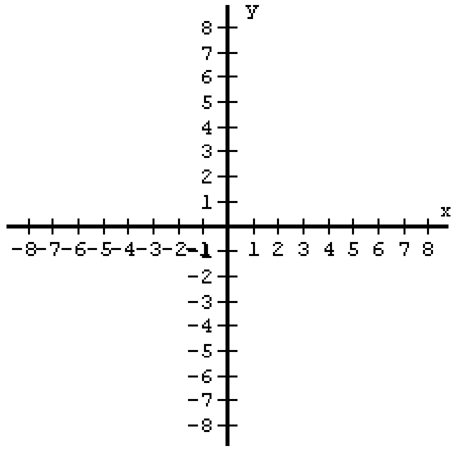


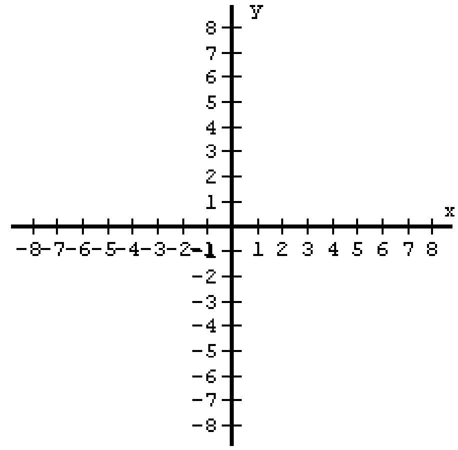
Equation of the Slant asymptote is: $y = x + 2$

Fact: The graph of a rational function will NEVER cross its vertical asymptote, but May cross its horizontal or slant asymptote. (see Case 1, the graph crosses its horizontal asymptote at $x = 2$)

ANALYZE RATIONAL FUNCTIONS



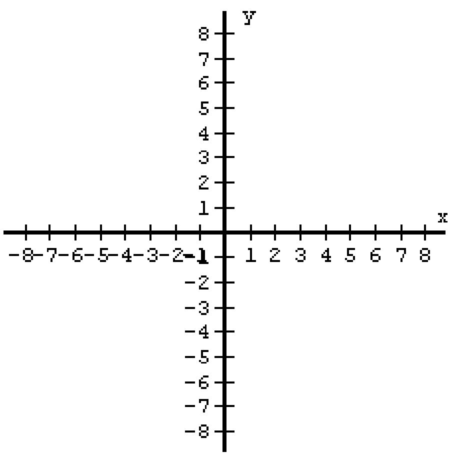


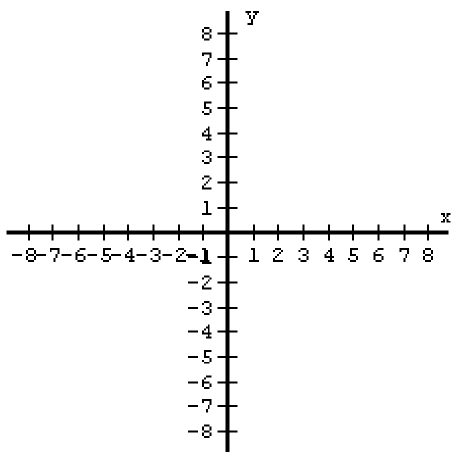


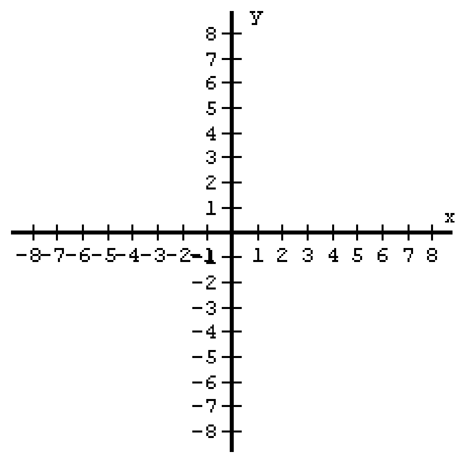
1) $y = \dfrac{1}{x-2}$

2) $y = \dfrac{x-1}{x-2}$

3) $y = \dfrac{(x-1)(x+3)}{(x-2)(x+4)}$



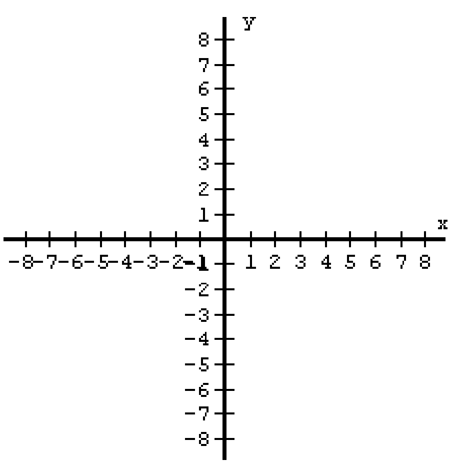


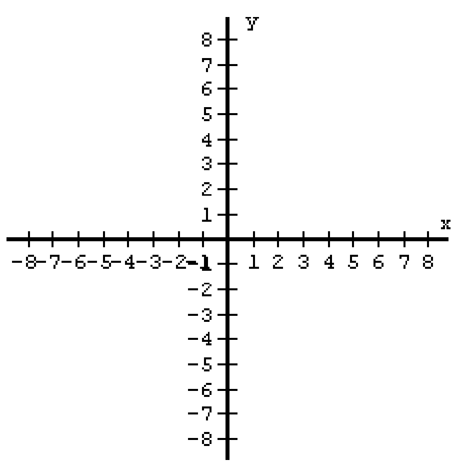


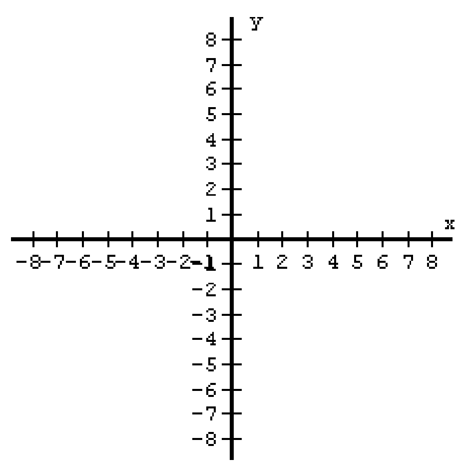
4) $y = \dfrac{x+1}{(x-2)^2}$

5) $y = \dfrac{(x+1)^2}{(x+2)(x-3)}$

6) $y = \dfrac{(x-1)^2}{(x+2)^2}$





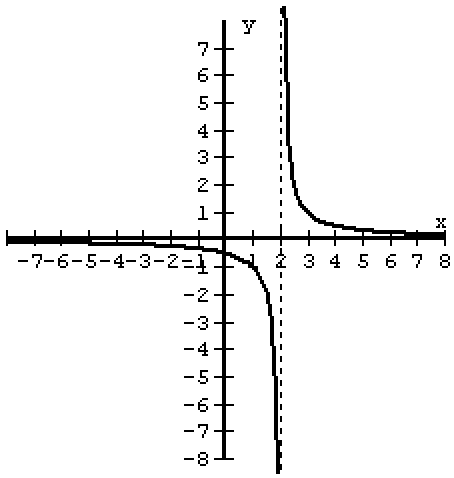


7) $y = \dfrac{(x-1)(x-3)}{(x-2)(x-3)}$

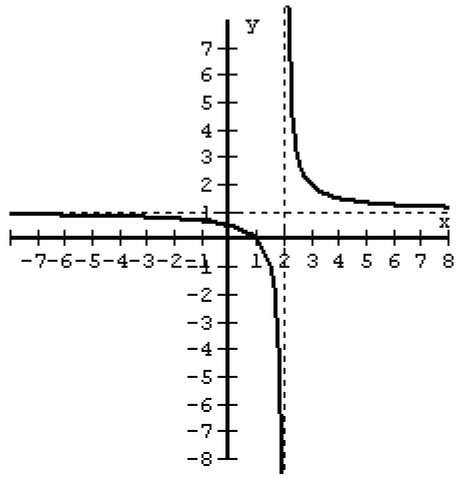
8) $y = \dfrac{(x-2)(x+3)}{x^2+4}$

9) $y = \dfrac{x^2+4}{(x-2)(x+3)}$

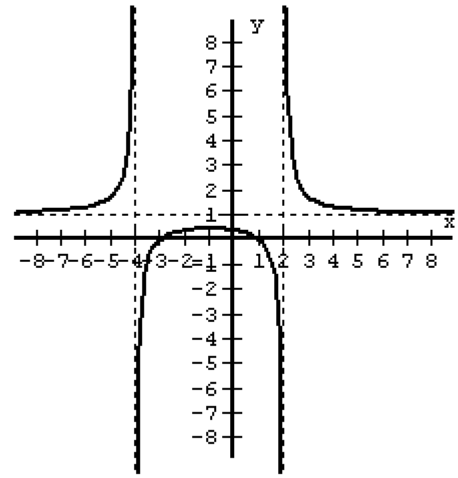
ANALYZE RATIONAL FUNCTIONS - KEY



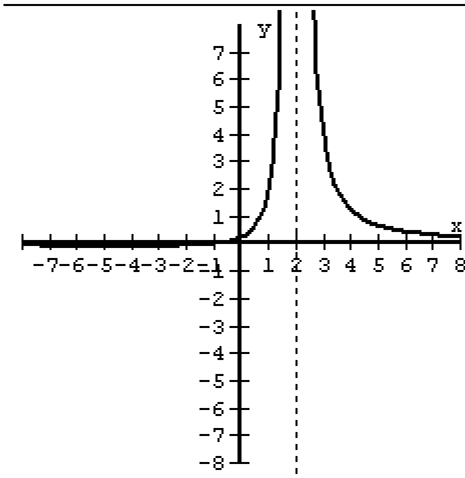
1) $y = \frac{1}{x - 2}$



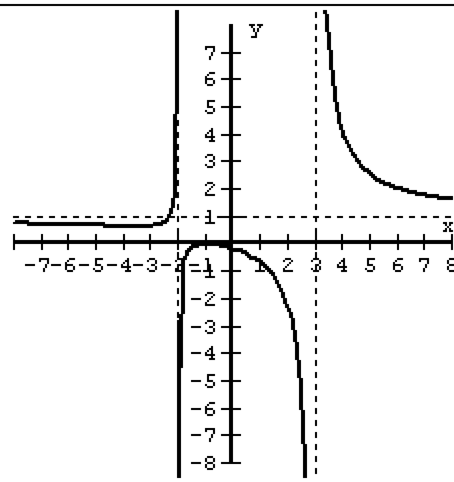
2) $y = \frac{x - 1}{x - 2}$



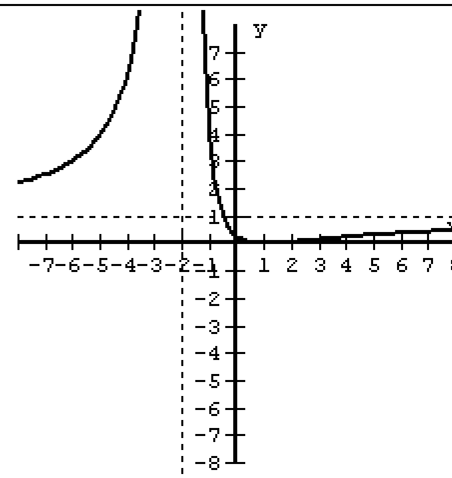
3) $y = \frac{(x-1)(x+3)}{(x-2)(x+4)}$



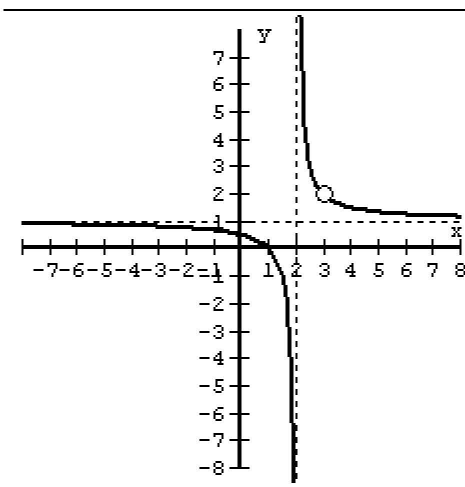
4) $y = \frac{x + 1}{(x - 2)^2}$



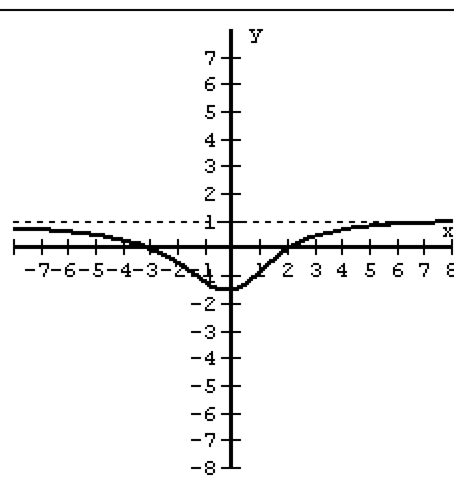
5) $y = \frac{(x+1)^2}{(x+2)(x-3)}$



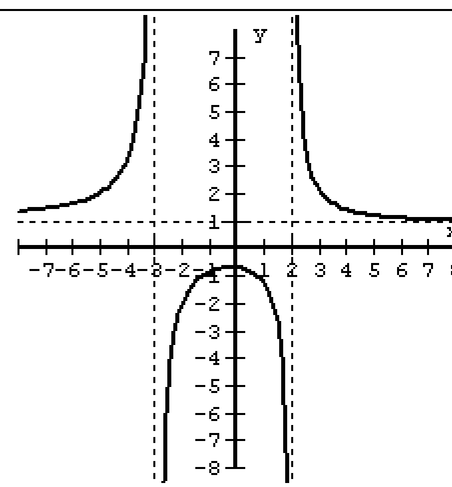
6) $y = \frac{(x-1)^2}{(x+2)^2}$



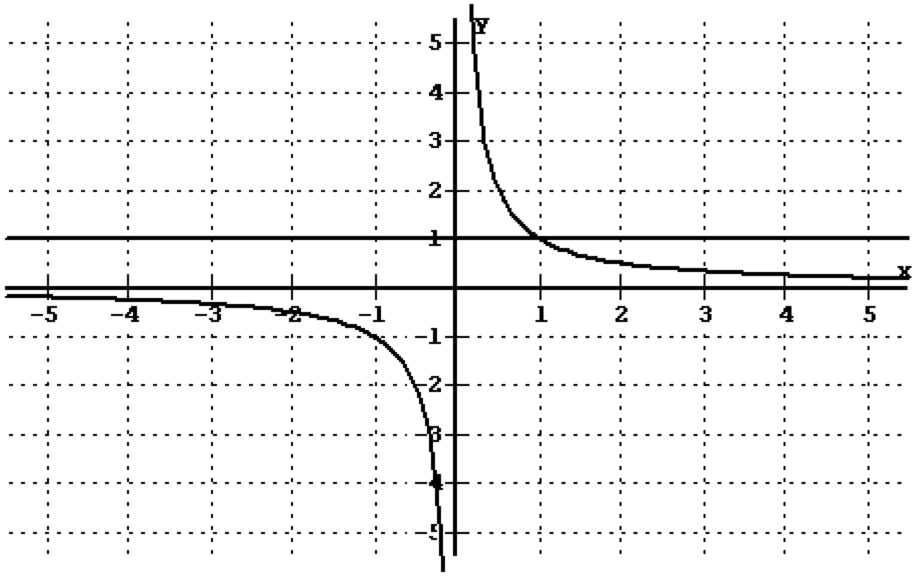
7) $y = \frac{(x-1)(x-3)}{(x-2)(x-3)}$



8) $y = \frac{(x-2)(x+3)}{x^2 + 4}$



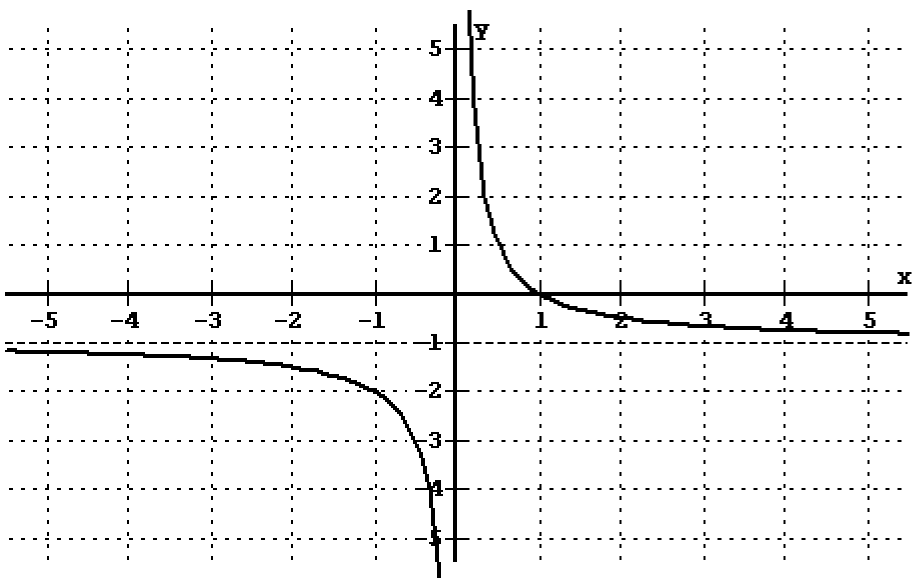
9) $y = \frac{x^2 + 4}{(x-2)(x+3)}$



Solve the rational inequality: $\frac{1}{x} \le 1$

Graphically,

Where is the graph of $y = \frac{1}{x}$ BELOW or ON the graph of $y = 1$?



Solve the rational inequality: $\frac{1}{x} - 1 \le 0$

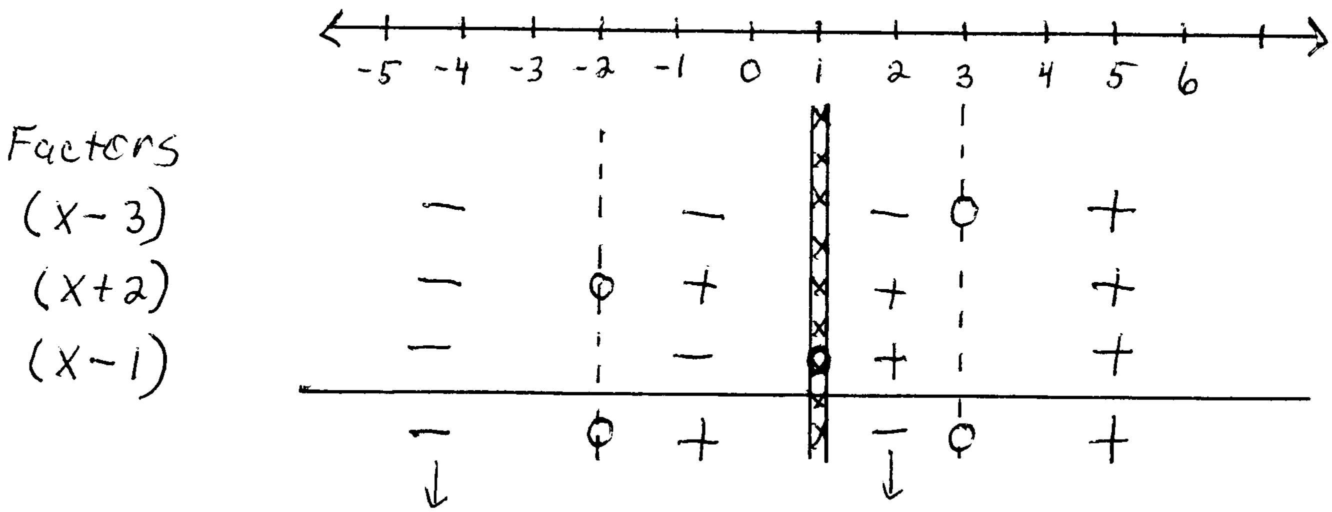
Graphically,

Where is the graph of $y = \frac{1}{x} - 1$ BELOW or ON the ***x*-axis**?

$$\frac{(x-3)(x+2)}{x-1} \le 0$$

Critical pts: $\frac{(x-3)(x+2)}{x-1}$

$x=3$, $x=-2$, $x \neq 1$

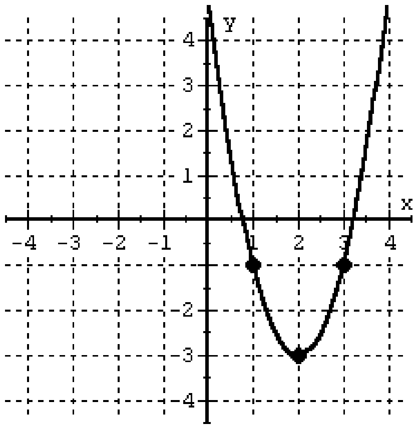


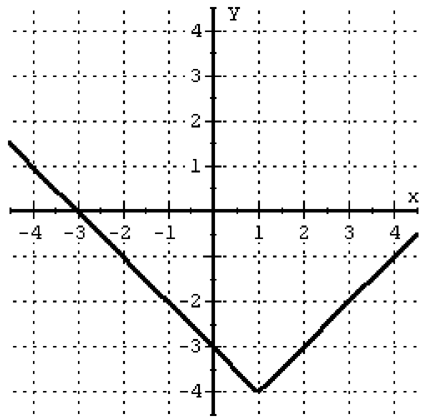
$$\frac{(x-3)(x+2)}{x-1} \le 0 \quad (-\infty, -2] \cup (1, 3]$$

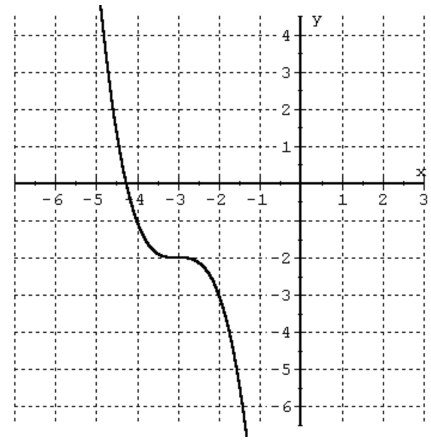
Negative or zero

MAC1140 Sample Test 1 Name ____________________

Use your knowledge of parent functions and transformations to find an equation of the following:



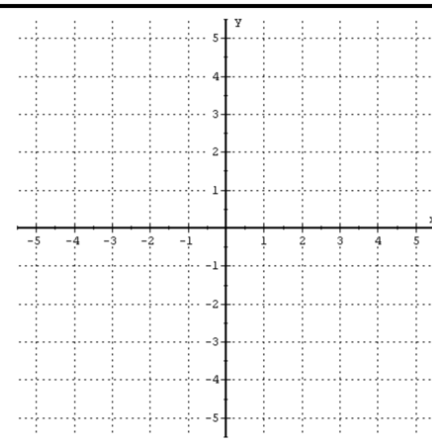




1) ____________________ 2) ____________________ 3) ____________________

---

4) Graph $f(x) = \begin{cases} x + 3 & \text{for } -4 \le x < 1 \\ -(x-2)^2 + 4 & \text{for } 1 \le x < 5 \end{cases}$



5) Let $f(x) = x^2 + 4$ and $g(x) = 3x - 4$. Find the following:

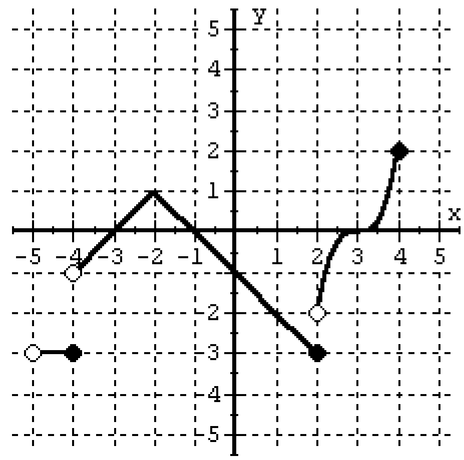
A) $2f(-3) + g(1)$

B) $(f \circ g)(5)$

C) Find the Difference Quotient $\dfrac{f(x+h) - f(x)}{h}$ of the function $f(x) = x^2 + 3x + 2$

6) Refer to the graph at the right.

A) Find a piecewise-defined function of the graph.



$f(x) = \begin{cases} \\ \\ \\ \end{cases}$

Find the following. Refer to the function above.

B) $\lim_{x \to -4^-} f(x)$

C) $\lim_{x \to -4^+} f(x)$

D) $\lim_{x \to -4} f(x)$

E) $f(-4)$

F) $f(-2)$

G) $\lim_{x \to 1} f(x)$

H) $\lim_{x \to 2^+} f(x)$

I) $\lim_{x \to 2^-} f(x)$

J) $f(2)$

---

7) **Find the following limits**

Let $f(x) = [\![x]\!] = \mathrm{int}(x)$ be the Greatest Integer function.

A) $\lim_{x \to 3^-} [\![x]\!]$

B) $\lim_{x \to 1^+} [\![x]\!]$

C) $\lim_{x \to -4^-} [\![x]\!]$

D) $\lim_{x \to 2} [\![x]\!]$

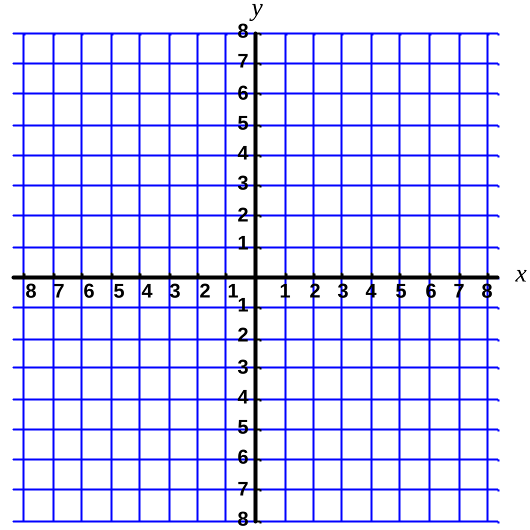
---

8) Solve the inequality: $\dfrac{x - 6}{x + 2} \geq -1$ Show your work. Express your answer in interval notation.

9) Use Substitution to solve $x^{\frac{2}{3}} - x^{\frac{1}{3}} - 6 = 0$ Show your work.

Sketch the following functions, showing all asymptotes and intercepts. **Give equations of all asymptotes**. Show your work.

10) $f(x) = \dfrac{3x - 4}{x + 2}$



Find the following Limits:

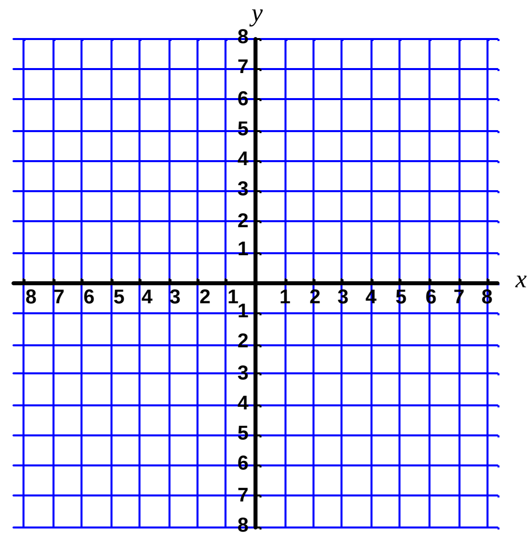
A) $\lim_{x \to -2^+} f(x)$

B) $\lim_{x \to -2^-} f(x)$

C) $\lim_{x \to +\infty} f(x)$

---

11) $f(x) = \dfrac{x - 4}{x^2 + 2x - 3}$



Find the following Limits:

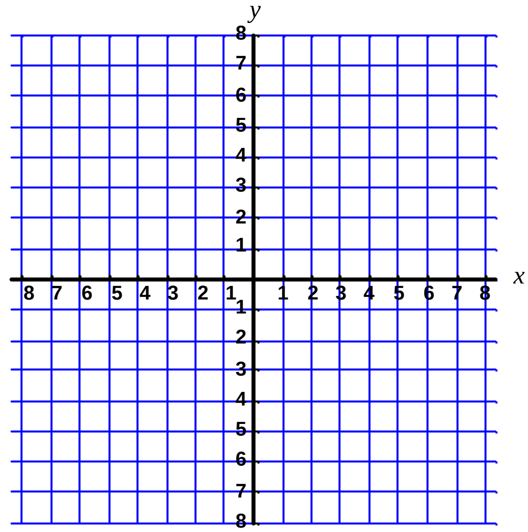
A) $\lim_{x \to -3^+} f(x)$

B) $\lim_{x \to 1^-} f(x)$

C) $\lim_{x \to -\infty} f(x)$

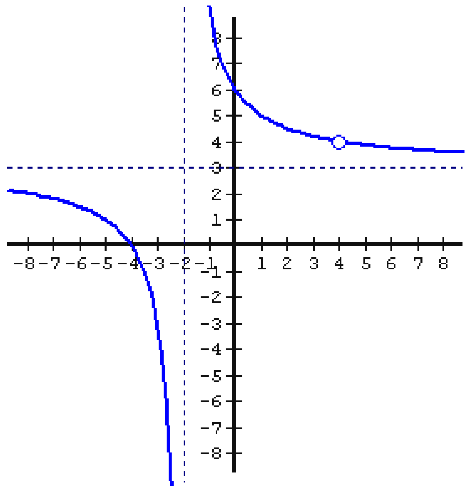
Sketch the following functions, showing all asymptotes and intercepts. **Give equations of all asymptotes**.

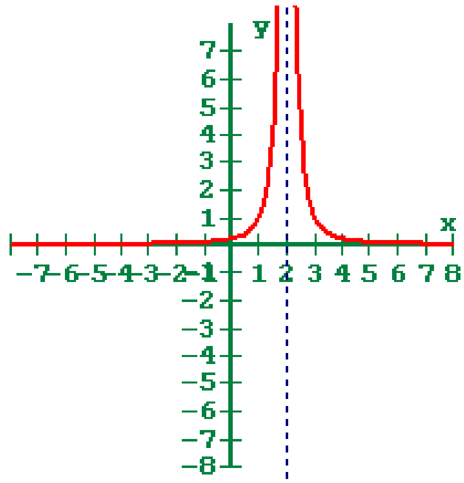
12) $f(x) = \dfrac{x^2 + 3x - 4}{x + 1}$



Find the following Limits:

A) $\lim\limits_{x \to -1^-} f(x)$ B) $\lim\limits_{x \to -4} f(x)$ C) $\lim\limits_{x \to +\infty} f(x)$





13) ______________________________
Write an equation for the graph above

A) $\lim\limits_{x \to -2^-} f(x)$

B) $\lim\limits_{x \to 4} f(x)$

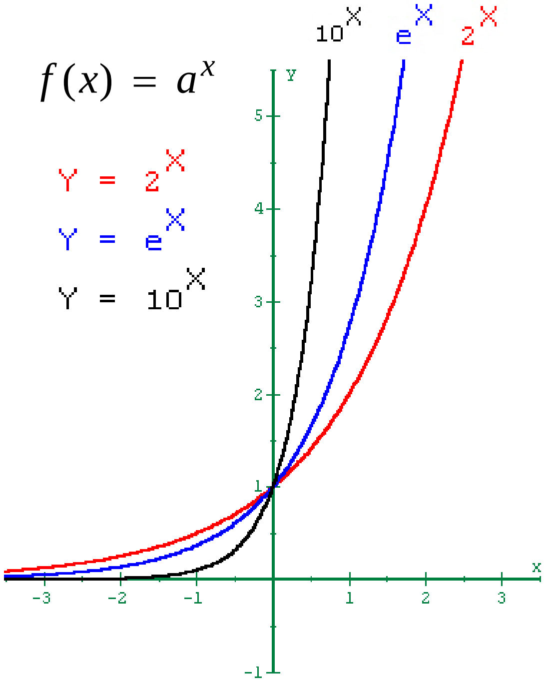
14) ______________________________
Write an equation for the graph above

A) $\lim\limits_{x \to 2^-} f(x)$

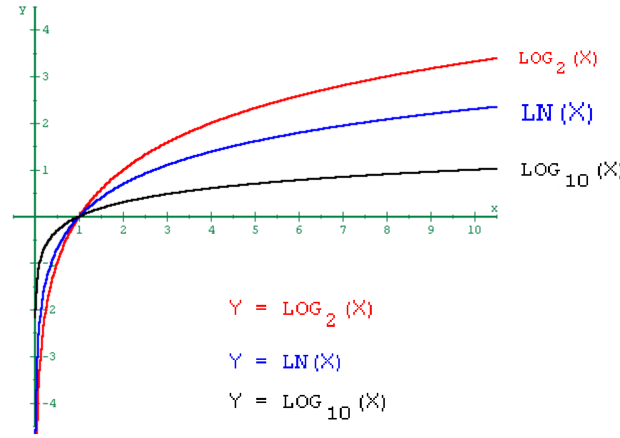
B) $\lim\limits_{x \to +\infty} f(x)$

Exponential and Logarithmic Functions Summary

$$f(x) = a^x$$



$$f^{-1}(x) = \log_a(x)$$



	$f(x)=a^x$	$f^{-1}(x)=\log_a(x)$
Domain:	$(-\infty, \infty)$	$(0, \infty)$
Range:	$(0, \infty)$	$(-\infty, \infty)$
x-intercept:	None	$x = 1$
y-intercept:	$y = 1$	None
Asymptote:	$y = 0$	$x = 0$

Converting logarithms to exponential form: $\log_a x = y \quad \Leftrightarrow \quad a^y = x$

Properties of Logarithms

I $\quad \log xy = \log x + \log y$

II $\quad \log \dfrac{x}{y} = \log x - \log y$

III $\quad \log x^n = n \log x$

Change of Base Formula

$$\log_a x = \frac{\log_b x}{\log_b a} \quad \text{ex)} \quad \log_2 x = \frac{\log_{10} x}{\log_{10} 2}$$

$$\log x = \log_{10} x \quad \text{and} \quad \ln x = \log_e x$$

$$\log \frac{x^3\sqrt{y}}{z^4} = 3\log x + \tfrac{1}{2}\log y - 4\log z$$

Solving Exponential Equations

$$5^x = 17$$

$$\log 5^x = \log 17$$

$$x \cdot \log 5 = \log 17$$

$$x = \frac{\log 17}{\log 5} \approx 1.76$$

Solving Logarithmic Equations

$$\log_2(x+5) = 3 \quad \Leftrightarrow \quad 2^3 = x + 5$$

$$8 = x + 5$$

$$x = 3$$

Compound Interest : $A = P\left(1 + \frac{r}{n}\right)^{nt}$

Population Growth : $A = Pe^{rt}$

One-to-One Functions

I Verify that f is a function.

Use the **Vertical Line Test** to show that f is a function:

A) A vertical line can pass through the graph of $y = f(x)$ at most once.

Alternately,

B) If a vertical line passes through the graph of $y = f(x)$ two or more times, then f is **not** a function; f is only a relation.

II Verify that the function f is one-to-one.

Once you have verified that f is a function,
use the **Horizontal Line Test** to show that f is one-to-one:

A) A horizontal line can pass through the graph of a one-to-one function $y = f(x)$ at most once.

Alternately,

B) If a horizontal line passes through the graph of the function $y = f(x)$ two or more times, then f is **not** one-to-one.

Important Facts

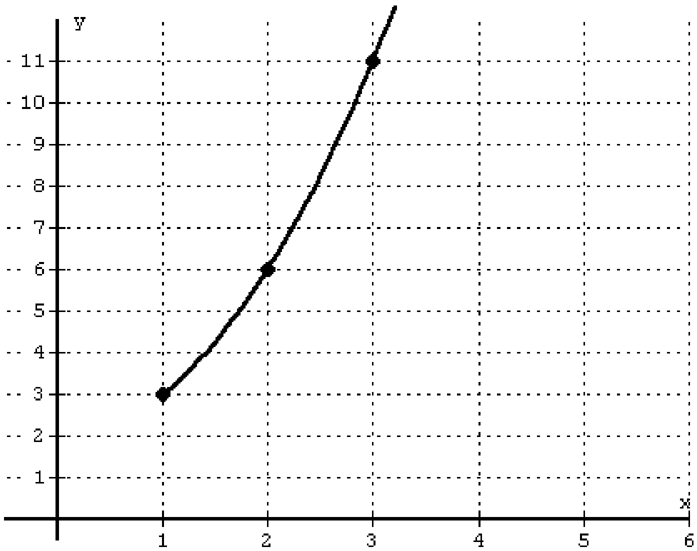
1) f has an inverse f^{-1} if and only if f is one-to-one.

That is to say,

If f is one-to-one, then f has an inverse f^{-1}, and
If f is not one-to-one, then f^{-1} does not exist.

2) The graphs of $y = f(x)$ and $y = f^{-1}(x)$ are symmetric about the graph of the line $y = x$.

Steps to Find the Inverse Function: $f^{-1}(x)$



$f(x) = x^2 + 2$ for $x \geq 1$

Domain f: $x \geq 1$

Range f: $y \geq 3$

Using the Horizontal Line Test, f is a one-to-one function, therefore $f^{-1}(x)$ exists.

Now find $f^{-1}(x)$.

$f(x)$		$f^{-1}(x)$	
$y = x^2 + 2$	$\rightarrow$	$\boldsymbol{x = y^2 + 2}$	Interchange the x's and y's
$x \geq 1$	$\rightarrow$	$y \geq 1$	
$y \geq 3$	$\rightarrow$	$x \geq 3$	

$x = y^2 + 2$ Solve for y.

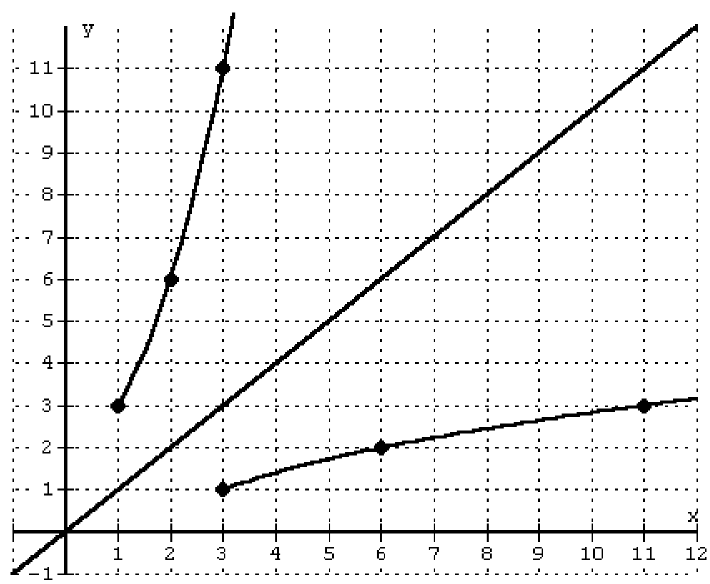
$x - 2 = y^2$

$\sqrt{x-2} = \sqrt{y^2}$

$y = \pm\sqrt{x-2}$ for $x \geq 3$, $y \geq 1$

Replace y with $f^{-1}(x)$

$f^{-1}(x) = \sqrt{x-2}$ for $x \geq 3$



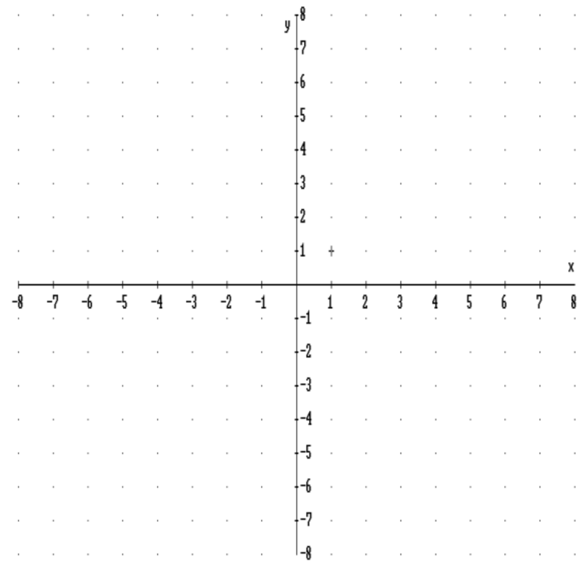
The graphs of $y = f(x)$ and $y = f^{-1}(x)$ are symmetric about the graph of the line $y = x$.

x	$f(x)$		x	$f^{-1}(x)$
1	3	$\leftrightarrow$	3	1
2	6	$\leftrightarrow$	6	2
3	11	$\leftrightarrow$	11	3

Swap the x and y values.

Functions and Inverses

Let $f(x) = (x+1)^3 + 2$ for $x \geq -3$



A) Graph $y = f(x)$

B) State the Domain of f

C) State the Range of f

D) Find $f^{-1}(x)$
(i.e. $f^{-1}(x) = \ldots$, including restrictions)

E) Graph $y = f^{-1}(x)$ on the grid above, using the appropriate restrictions.

F) State the Domain of $f^{-1}(x)$

G) State the Range of $f^{-1}(x)$

H) Graph the dotted line: $y = x$
Compare the graphs of $f(x)$ and $f^{-1}(x)$ for symmetry, both numerically and graphically.

How do you prove that a relation is a function?

For each x-value, There is only 1 y-value

If (a, b) and (a, c) are points on the relation, then $b = c$.

---

ex) $y = 5x + 2$

$(a, b) \rightarrow \quad b = 5a + 2$
$(a, c) \rightarrow \quad c = 5a + 2$

$b = 5a + 2 = c$
$b = c$ ✓
$\therefore y = 5x + 2$ is a fcn

---

ex) $x^2 + y^2 = 9$

$(a, b) \quad a^2 + b^2 = 9$
$(a, c) \quad a^2 + c^2 = 9$ $\rightarrow$

$a^2 + b^2 = a^2 + c^2$
$-a^2 \qquad -a^2$
$\sqrt{b^2} = \sqrt{c^2}$
$b = \pm c$
$b \neq c$
$\therefore x^2 + y^2 = 9$ is Not a fcn

One-to-One fcn - Algebraic Proof

Given an x-value, there is only 1 y-value

A function f is one-to-one,
if for a and b in the Domain of f,
$f(a) = f(b) \Rightarrow a = b$

1-1 fcn: Given a y-value, there is only 1 x-value

---

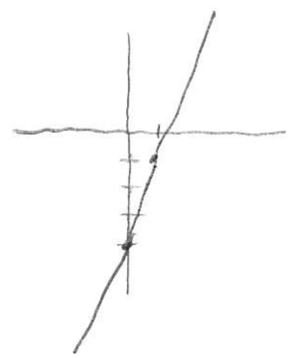
ex) Show that $f(x) = 3x - 4$ is one-to-one

$$f(x) = 3x - 4$$

Assume: $f(a) = f(b)$

(Show $a = b$)

$$3(a) - 4 = 3(b) - 4$$
$$3a - 4 = 3b - 4$$
$$3a = 3b$$
$$a = b \checkmark$$



✓

---

ex) Show that $f(x) = x^2 + 2$ is Not one-to-one

Assume: $f(a) = f(b)$

(Show $a \neq b$)

$$a^2 + 2 = b^2 + 2$$
$$\sqrt{a^2} = \sqrt{b^2}$$
$$|a| = |b|$$
$$\pm a = \pm b$$

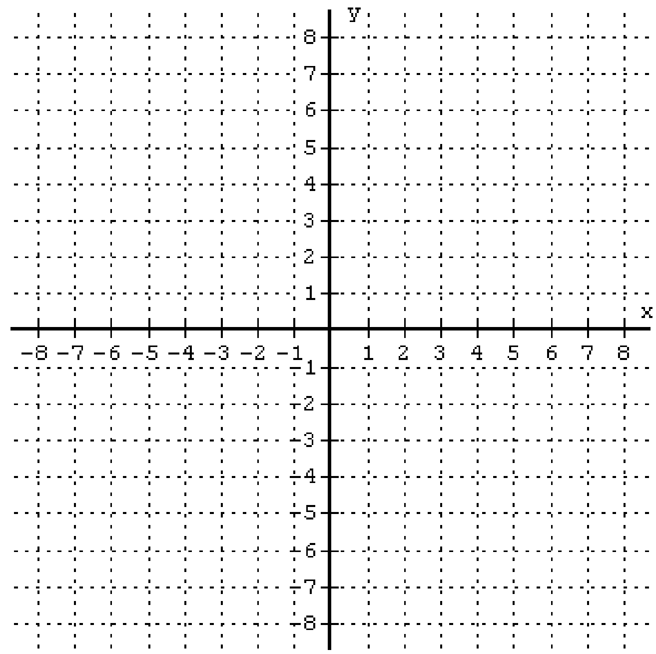
$a = b$ or $a = -b$

$\therefore$ Not 1-1

______

Analyze Graphs

1) Equation of Graph



2) Calculator Notation

3) What Type of Graph is this?

4) Function? yes/no

5) One-to-One Function? yes/no

6) State any Symmetry:

7) Domain:

8) Range:

9) *x*-intercept(s):

10) *y*-intercept:

11) Where is $f(x) < 0$?
State the x-values using interval notation

12) Where is $f(x) \geq 0$?
State the x-values using interval notation

13) Where does $f(x) = 0$?
List the x-value(s)

14) Where is $f(x)$ increasing?
State the x-values using interval notation

15) Where is $f(x)$ decreasing?
State the x-values using interval notation

16) What is the Absolute Maximum value?

17) What is the Absolute Minimum value?

18) Are there any Asymptotes?
If yes, then give the equation(s)

Compound Interest

$$A = P\left(1 + \frac{r}{n}\right)^{(n \cdot t)}$$

Amount (Final Balance: Principal + Interest)

Principal (Starting amount)

rate (as a decimal)

number (number of times compounded per year)

time (in years)

Ex) Invest $800 at 5% annual interest, compounded daily, for 6 years.

$$A = \$800\left(1 + \frac{0.05}{365}\right)^{(365 \times 6)} = \$1079.86$$

$$A = 800\left(1 + 0.05 / 365\right) \wedge (365 * 6) = \$1079.86$$

Ex) Suppose that you invest $1000 in an account that earns 8% annual interest. Find the balance in the account at the end of 10 years if your money is compounded:

Compounded	Value of n	Calculator Notation	Amount
Simple Interest	N/A	Use the formula: $A = P + Prt$	$
Yearly	$n = 1$	1000 (1 + 0.08 / 1) ^ (1 * 10)	$
Quarterly	$n = 4$		$
Monthly	$n = 12$		$
Daily	$n = 365$		$
Continuously!!!	$n \to \infty$	Use the formula: $A = Pe^{rt}$	$

Monthly Payment on a Loan - Formula

P = the amount borrowed (the **P**rincipal)

r = the <u>annual</u> interest **r**ate

n = the **n**umber of <u>months</u> of the loan

M = the **m**onthly payment

$$M = \frac{P\left(1+\frac{r}{12}\right)^n\left(\frac{r}{12}\right)}{\left(1+\frac{r}{12}\right)^n - 1}$$

To make it easier to enter into your calculator:

$$M = \left(P\ (1 + r/12)\ ^\wedge\ n\ *\ (r/12)\right) / \left(\ (\ (1 + r/12)\ ^\wedge\ n - 1\ )\right)$$

Example)

Suppose you need to get a loan to buy a used truck. You borrow $7,000 for 4 years at 15% annual interest. Find your monthly payment.

P = $7,000
r = 15% = 0.15
n = 4 x 12 = 48
M = Your monthly payment

$$M = \left(7000\,(1 + 0.15/12)\ ^\wedge\ 48 * (0.15/12)\right) / \left(\,(1 + 0.15/12)\ ^\wedge\ 48 - 1\right)$$

$$M = \$194.82$$

Simplifying Logarithms

$\log_{2} 8 = \square \leftrightarrow 2^{\square} = 8$ check: $\log_2 8 = 3 \leftrightarrow 2^3 = 8$

To what power do you raise 2 to get 8?

$\log_5 25 = \square \leftrightarrow 5^{\square} = 25$

$\log_3 81 = \square \leftrightarrow 3^{\square} = 81$

$\log_4 16 = \square \leftrightarrow 4^{\square} = 16$

$\log_6 1 = \square \leftrightarrow 6^{\square} = 1$

$\log_2 \left(\frac{1}{8}\right) = \square \leftrightarrow 2^{\square} = \frac{1}{8}$

$\log 100 = \square \leftrightarrow ?^{\square} = 100$

$\log 1000 = \square$

$\log 0 = \square$

$\log(-10) = \square$

$\log_2 6 \approx$

$\log 250 \approx$

How were logarithms used in the past?

Before there were calculators, mathematicians, scientists, astronomers and engineers used logarithms to multiply, divide and find roots of numbers.

Here's the principal behind using logarithms.

$$16 \times 8 = 128$$

$$\underbrace{2^4 \times 2^3}_{} = 2^{4+3} = 2^7 = 128$$

bases are the same, so combine using the laws of exponents

$$125 \times 81$$

$$\underbrace{5^3 \times 3^4}_{}$$

bases are NOT the same, so the laws of exponents do not apply.

$$125 \times 81 = 10{,}125$$

$$\underbrace{10^{2.096910} \times 10^{1.908485}}_{}$$

bases are the same, so combine using the laws of exponents.

$$\log 125 \approx 2.096910$$

$$\log 81 \approx 1.908485$$

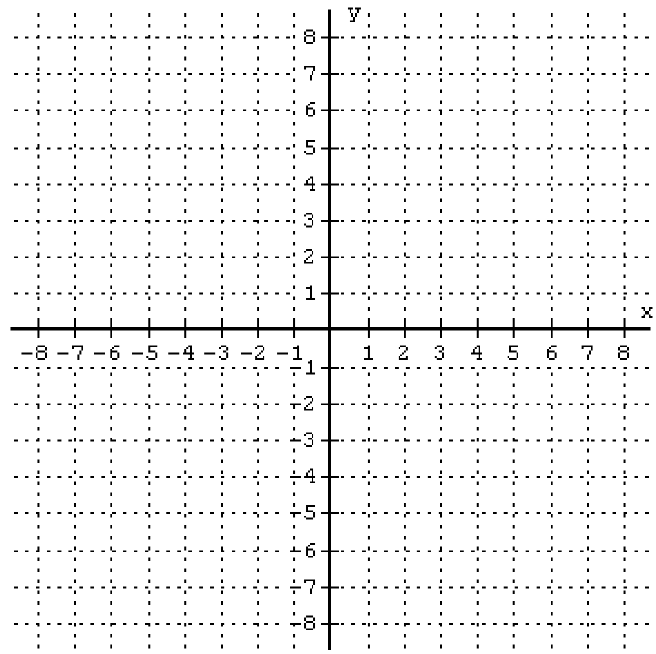
$$= 10^{2.096910+1.908485}$$

$$= 10^{4.005395}$$

$$= 10{,}124.99926 \approx 10{,}125$$

______

Analyze Graphs



1) Equation of Graph

2) Calculator Notation

3) What Type of Graph is this?

4) Function? yes/no

5) One-to-One Function? yes/no

6) State any Symmetry:

7) Domain:

8) Range:

9) *x*-intercept(s):

10) *y*-intercept:

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16) What is the Absolute Maximum value?

17) What is the Absolute Minimum value?

18) Are there any Asymptotes?
If yes, then give the equation(s)

Drug Medication

The formula $D(t) = D_0\, e^{-0.2t}$ can be used to find the number of milligrams (mg) ***D*** of a certain drug that is in a patient's bloodstream after ***t*** hours, assuming that $\boldsymbol{D_0}$ mg of the drug is administered initially (t = 0). Assume 5 mg of the drug is administered initially.

Equation: ________________________________

1. Approximately how many mg of the drug will be present in the bloodstream after …

0 hours: Calculator entry: ____________________________ ≈ __________ mg
(Round to 1 decimal place)

1 hour: Calculator entry: ____________________________ ≈ __________ mg

3 hours: Calculator entry: ____________________________ ≈ __________ mg

6 hours: Calculator entry: ____________________________ ≈ __________ mg

10 hours: Calculator entry: ____________________________ ≈ __________ mg

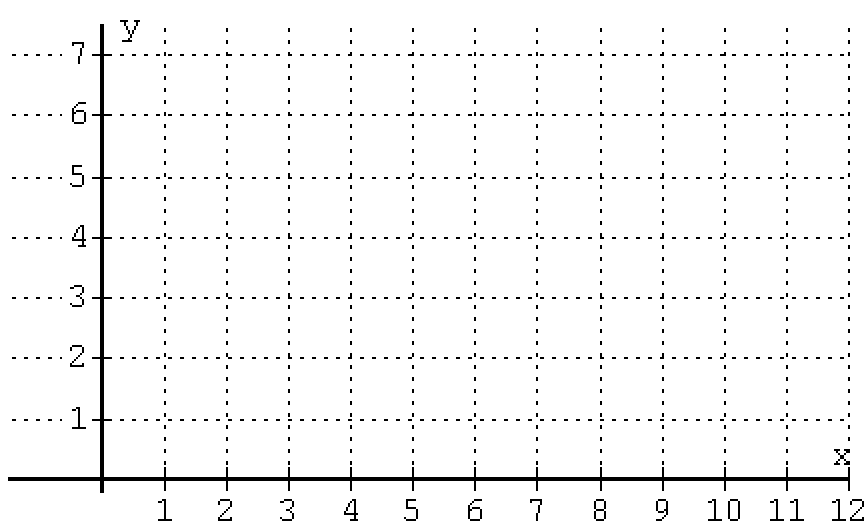
2. Use this data and sketch the graph of the exponential function below. Label the axes.

3. When the number of mg of the drug in the patient's bloodstream reaches 2 mg, the drug needs to be administered again. Approximately how long until another injection needs to be administered? (Round your answer to 1 decimal place)

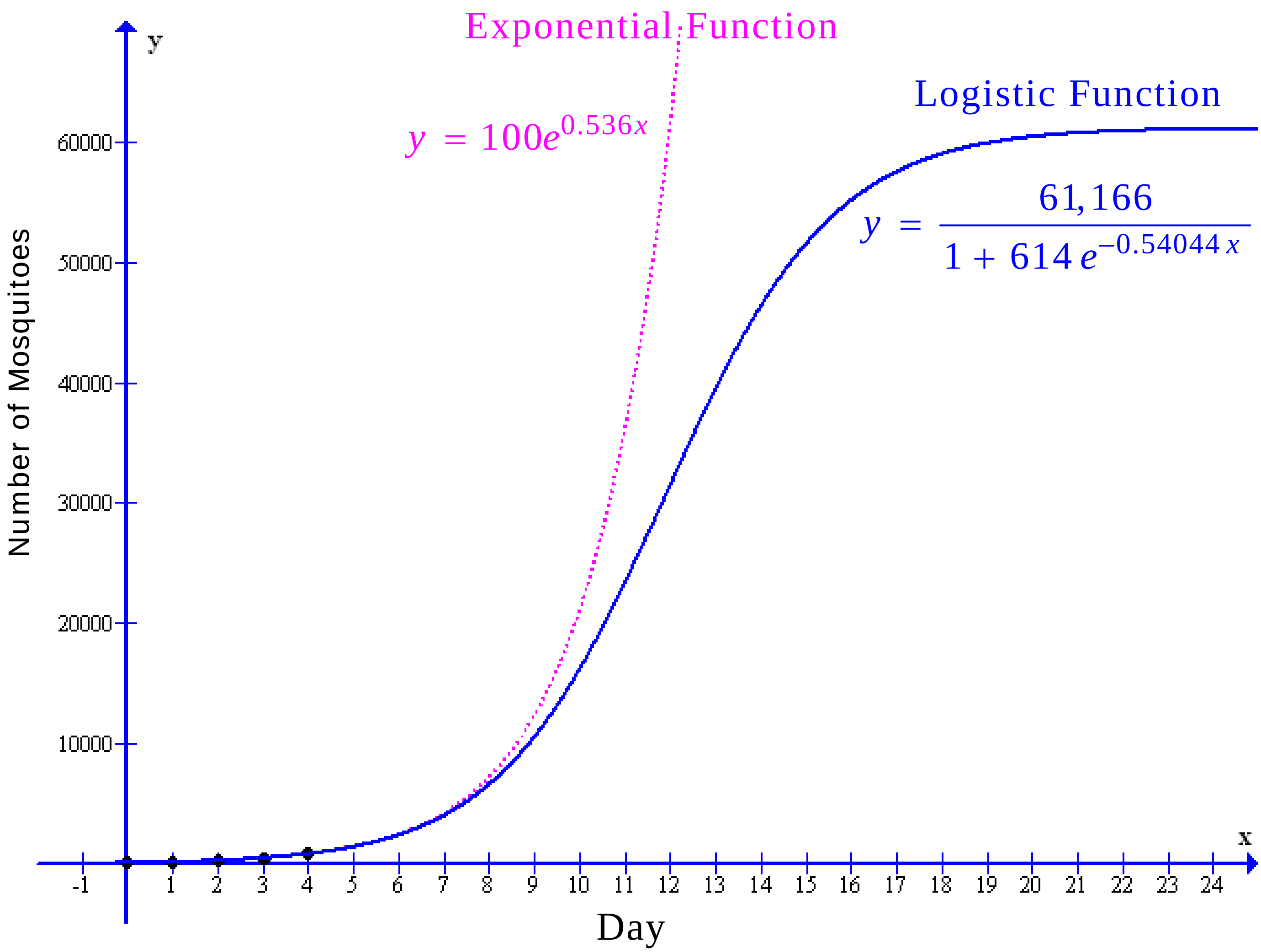
Solve Algebraically

Equation:

Solve Graphically



Exponential Function vs. Logistic Function



Day 0: 100 mosquitoes (initial population)
Day 3: 500 mosquitoes

Resulting exponential function:

$$y = 100e^{0.536x}$$

Logistic Function of best fit:

$$y = \frac{61{,}166}{1 + 614\,e^{-0.54044\,x}}$$

x	y	y
0	100	100
1	170	170
2	292	292
3	500	500
4	853	853
10	21,272	16,260
20	4,530,000	60,416

MAC1140 Sample Test 2

Name ___________________

1) Let $f(x) = \sqrt{3x - 5}$. Find $f^{-1}(x)$.

---

2) Suppose you invest $30,000 in a CD that earns 4.5% annual interest, compounded daily. Find the amount in the account at the end of 7 years. Recall the formula: $A = P\left(1 + \frac{r}{n}\right)^{(n \cdot t)}$

---

3) Monthly Payment on a Loan - Formula

P = the amount borrowed (the **P**rincipal)
r = the annual interest **r**ate
n = the **n**umber of months of the loan
M = the **m**onthly payment

$$M = \frac{P\left(1 + \frac{r}{12}\right)^n \left(\frac{r}{12}\right)}{\left(1 + \frac{r}{12}\right)^n - 1}$$

To make it easier to enter into your calculator:

$$M = \big(P\,(1 + r/12) \wedge n \;\; * \;\; (r/12)\big) / \big(((1 + r/12) \wedge n - 1)\big)$$

Suppose that you want to purchase a new home costing $125,000. You take out a 30 year mortgage at 5% annual percentage rate. Find your monthly payment.

4) Use your calculator and the Change-of-Base Formula to evaluate the following logarithm:

$\log_2 50$

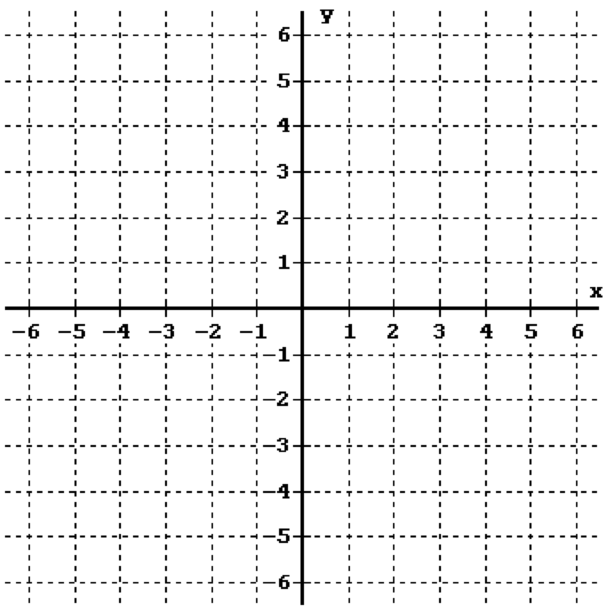
5) Expand the following logarithms to the Sum or Difference of Logarithms. Simplify Completely.

$$\log \frac{w^2 \sqrt{x}}{y^3 \; z^4}$$

6) Condense the following logarithmic expressions to a single logarithm. Simplify Completely.

$\frac{1}{3} \log x - 2 \log y - 4 \log z$

7)

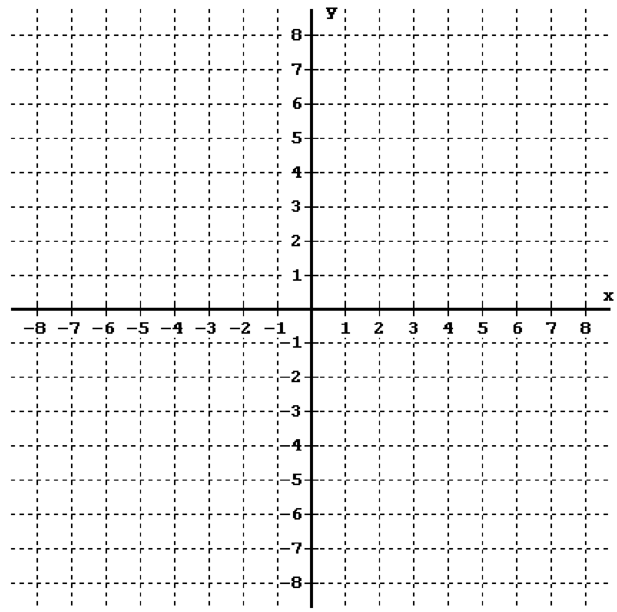


A) Graph: $f(x) = 2^{x-3} - 4$

B) State the Domain

C) State the Range

8)



A) Graph: $f(x) = \log_2(x + 5)$

B) State the Domain

C) State the Range

9) The formula $D = D_0 e^{-0.3t}$ can be used to find the number of milligrams (mg) D of a certain drug that is in a patient's bloodstream t hours after the drug has been administered where D_0 is the initial amount of the drug administered. Assume that 10 mg of the drug is administered initially.

A) Use your calculator to determine how much of the drug is in the patient's bloodstream 3 hours after it was administered. Round your answer to 1 decimal place.

B) When the amount of the drug that is in a patient's bloodstream reaches 5 mg, the drug needs to be re-administered. How long until the drug needs to be re-administered? **Solve by hand**. Show your work. Round your answer to 1 decimal place.

Solve the following equations algebraically, then use your calculator to round your answers to 2 decimal places.

10) $\log_2(x + 3) = 4$

11) $\ln(2x - 5) = 3$

12) $2^{x+4} = 6$

13) $e^{x-4} = 16$

---

14) Solve $A = P\left(1 + \frac{r}{n}\right)^{nt}$ for $\boldsymbol{t}$

Bonus: Solve $y = \dfrac{k}{1 + ae^{-bx}}$ for x.