Infinite v.s. the Infinitesimal

Estimating the Area Under a Curve - Limits

**Integral**

<table>
<thead>
<tr>
<th>Number of Rectangles</th>
<th>Estimated Area: Riemann Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
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</tr>
<tr>
<td>4</td>
<td>18</td>
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<td>6</td>
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<tr>
<td>8</td>
<td>18.5</td>
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<tr>
<td>10</td>
<td>18.56</td>
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<tr>
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<td>18.64</td>
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<tr>
<td>40</td>
<td>18.66</td>
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<tr>
<td>100</td>
<td>18.6656</td>
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<tr>
<td>500</td>
<td>18.66662</td>
</tr>
<tr>
<td>1000</td>
<td>18.66666</td>
</tr>
</tbody>
</table>

Infinite: Not finite; having a value greater than any computable value.

Infinitesimal: Approaching zero as a limit; arbitrarily small; infinitely small.

The Limit of the Estimated Area, using Riemann Sums, is $18\frac{2}{3}$

Using the Definite Integral in Calculus,

\[ \text{Area} = \int_{-1}^{3} -(x - 1)^2 + 6 \, dx = 18\frac{2}{3} \]