$\lim_{x \to 0^+} \frac{1}{x}$ does not exist

However, we write: $\lim_{x \to 0^+} \frac{1}{x} = \infty$

$y = f(x) = \frac{1}{x}$ has no limit as $x \to 0^+$. It is nevertheless convenient to describe the behavior of $f$ by saying that $f(x)$ approaches $\infty$ as $x \to 0^+$.

When we write $\lim_{x \to 0^+} \frac{1}{x} = \infty$, we are not saying that the limit exists.

We are not saying that there is a real number $\infty$, for there is no such number.

Rather, we are saying that $\lim_{x \to 0^+} \frac{1}{x}$ does not exist because $\frac{1}{x}$ becomes arbitrarily large and positive as $x \to 0^+$.

(Extracted from Thomas' Calculus Early Transcendentals Media Upgrade, 11th ed. page 110)