Continuous Functions - Definitions

Continuity (Informal Definition)

A function is **continuous** over an interval of its domain if its hand-drawn graph over that interval can be sketched without lifting the pencil from the paper.

Continuity (Limit Definition)

A function \( f \) is continuous at \( x = a \) if and only if \( \lim_{x \to a} f(x) = f(a) \); that is, \( \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a) \).

Continuity: (\( \varepsilon, \delta \) Definition)

A function \( f \) is continuous at \( x = a \) if and only if

\[
\forall \varepsilon > 0, \exists \delta > 0, \text{ such that } |f(x) - f(a)| < \varepsilon \quad \text{if} \quad |x - a| < \delta .
\]