DEFINITION OF CONTINUITY

A function \( f \) is said to be continuous at \( x = c \) provided the following conditions are satisfied:

1. \( f(c) \) is defined
2. \( \lim_{{x \to c}} f(x) \) exists
3. \( \lim_{{x \to c}} f(x) = f(c) \)

\[ f \text{ is continuous at } x = c \iff \lim_{{x \to c}} f(x) = f(c) \]

Note: The definition above implies the following:

\[ \lim_{{x \to c^-}} f(x) = \lim_{{x \to c^+}} f(x) = f(c) \]  \( \iff \)  \( \lim_{{x \to c}} f(x) \) exists  \( \iff \)  \( f(c) \) is defined  \( \iff \)  \( f(c) \) exists

Theorems

Suppose the functions \( f \) and \( g \) are continuous at \( c \), then

- \( f \pm g \) are continuous at \( c \).
- \( fg \) is continuous at \( c \).
- \( f / g \) is continuous at \( c \) provided \( g(c) \neq 0 \).
- \( f \circ g \) is continuous at \( c \) provided \( g \) is continuous at \( c \) and \( f \) is continuous at \( g(c) \).

A polynomial is continuous everywhere.

A rational function is continuous at every point where the denominator is nonzero.