Differentiability $\Rightarrow$ Continuity

\[ f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c} = L \quad \Rightarrow \quad \lim_{x \to c} f(x) = f(c) \]

**Proof**

\[ \lim_{x \to c} \left[ f(x) - f(c) \right] = \lim_{x \to c} \left[ (x - c) \cdot \frac{f(x) - f(c)}{x - c} \right] \text{ for } x \neq c \]

\[ = \lim_{x \to c} (x - c) \cdot \lim_{x \to c} \frac{f(x) - f(c)}{x - c} \]

**note:** you can make the product of 2 limits only if each limit exists

\[ = 0 \times f'(c) \]

The derivative exists by Hypothesis $f'(c) \to \pm \infty$

\[ \lim_{x \to c} \left[ f(x) - f(c) \right] = 0 \]

\[ \lim_{x \to c} f(x) - \lim_{x \to c} f(c) = 0 \]

\[ \lim_{x \to c} f(x) = \lim_{x \to c} f(c) \]

\[ \lim_{x \to c} f(x) = f(c) \checkmark \text{ is the Def. for Continuity} \]

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Note: $f$ is Continuous $\not\Rightarrow f$ is Differentiable

**Proof, by Counterexample**

Let $f(x) = |x|$ is continuous at $x = 0$

But $f(x) = |x|$ is Not differentiable at $x = 0$

Left slope $= -1 \neq$ Right slope $= 1$