

## Gaussian Elimination

$$\begin{array}{rcl}
 3x + 5y - z = -7 & & 3x + 5y - 1z = -7 \\
 x + y + z = -1 & \rightarrow & 1x + 1y + 1z = -1 \rightarrow \\
 2x + y + 11z = 7 & & 2x + 1y + 11z = 7
 \end{array}$$

$$\begin{bmatrix} 3 & 5 & -1 & -7 \\ 1 & 1 & 1 & -1 \\ 2 & 1 & 11 & 7 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 & -1 \\ 3 & 5 & -1 & -7 \\ 2 & 1 & 11 & 7 \end{bmatrix} \xrightarrow{\boxed{-3}} \begin{bmatrix} 1 & 1 & 1 & -1 \\ 0 & 2 & -4 & -4 \\ 2 & 1 & 11 & 7 \end{bmatrix} \xrightarrow{\boxed{\frac{1}{2}}} \rightarrow$$

$\uparrow$  Swap  $R_1$  and  $R_2$        $\uparrow$   $R_2 = -3R_1 + R_2$

$$\begin{bmatrix} 1 & 1 & 1 & -1 \\ 0 & 1 & -2 & -2 \\ 2 & 1 & 11 & 7 \end{bmatrix} \xrightarrow{\boxed{-2}} \begin{bmatrix} 1 & 1 & 1 & -1 \\ 0 & 1 & -2 & -2 \\ 0 & -1 & 9 & 9 \end{bmatrix} \xrightarrow{\boxed{1}} \begin{bmatrix} 1 & 1 & 1 & -1 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & 7 & 7 \end{bmatrix} \xrightarrow{\boxed{\frac{1}{7}}} \rightarrow$$

$\uparrow$   $R_2 = \frac{1}{2}R_2$        $\uparrow$   $R_3 = -2R_1 + R_3$        $\uparrow$   $R_3 = R_2 + R_3$

x	y	z			
$\begin{bmatrix} 1 & 1 & 1 & -1 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$	$\rightarrow$	$\rightarrow$	$1x + 1y + 1z = -1$		
	$\rightarrow$	$1y - 2z = -2$	$x + 1(0) + 1(1) = -1$		
	$1z = 1$	$y - 2(1) = -2$	$x = -2$		
$\uparrow$ $R_3 = \frac{1}{7}R_3$	$z = 1$	$y = 0$			

$\underbrace{\hspace{15em}}$   
Row Echelon Form

Using Back-Substitution:  $x = -2$  ,  $y = 0$  ,  $z = 1$

## Without Using Back-Substitution:

$$\begin{bmatrix} 1 & 1 & 1 & -1 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{\boxed{2}} \begin{bmatrix} 1 & 1 & 1 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{\boxed{-1}} \begin{bmatrix} 1 & 1 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{\boxed{-1}} \rightarrow$$

$\underbrace{\hspace{10em}}_{\text{Row Echelon Form}} \quad \uparrow R_2 = 2R_3 + R_2 \quad \uparrow R_1 = -1R_3 + R_1$

x   y   z

$$\begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \rightarrow \begin{array}{l} x = -2 \\ y = 0 \\ z = 1 \end{array}$$

$$\uparrow R_1 = -1R_2 + R_1$$

$\underbrace{\hspace{10em}}_{\text{Reduced}}$

Row Echelon Form