HISTORY OF LOGARITHMS

*Joost Bürgi*, a Swiss clockmaker in the employ of the Duke of Hesse-Kassel, first conceived of logarithms.

The method of natural logarithms was first propounded in *1614*, in a book entitled *Mirifici Logarithmorum Canonis Descriptio*, by *John Napier*, *Baron of Merchiston* in *Scotland*, four years after the publication of his memorable invention. This method contributed to the advance of science, and especially of astronomy, by making some difficult calculations possible. Prior to the advent of calculators and computers, it was used constantly in surveying, navigation, and other branches of practical mathematics.

At first, Napier called logarithms "artificial numbers" and antilogarithms "natural numbers". Later, Napier formed the word *logarithm*, a *portmanteau*, to mean a number that indicates a ratio: ^o°o? (logos) meaning ratio, and a??µo? (arithmos) meaning number. Napier chose that because the difference of two logarithms determines the ratio of the numbers for which they stand, so that an arithmetic series of logarithms corresponds to a geometric series of numbers. The term antilogarithm was introduced in the late 17th century and, while never used extensively in mathematics, persisted in collections of tables until they fell into disuse.

Napier did not use a base as we now understand it, but his logarithms were, up to a scaling factor, effectively to base 1/e. For interpolation purposes and ease of calculation, it is useful to make the ratio $r$ in the geometric series close to 1. Napier chose $r=1-10^-7=0.999999$, and Bürgi chose $r=1+10^-4=1.0001$. Napier’s original logarithms did not have log 1 = 0 but rather log 10^-7 = 0. Thus if $N$ is a number and $L$ is its logarithm as calculated by Napier, $N=10^7(1-10^-7)^L$. Since $(1-10^-7)$ is approximately 1/e, $L$ is approximately $10^7log_{1/e} N/10^7$. 
Tables of logarithms

Prior to the advent of computers and calculators, using logarithms meant using tables of logarithms, which had to be created manually. Base-10 logarithms are useful in computations when electronic means are not available. See common logarithm for details, including the use of characteristics and mantissas of common (i.e., base-10) logarithms.

In 1617, Briggs published the first installment of his own table of common logarithms, containing the logarithms of all integers below 1000 to eight decimal places. This he followed, in 1624, by his Arithmetica Logarithmica, containing the logarithms of all integers from 1 to 20,000 and from 90,000 to 100,000 to fourteen places of decimals, together with a learned introduction, in which the theory and use of logarithms are fully developed. The interval from 20,000 to 90,000 was filled up by Adrian Vlacq, a Dutch computer; but in his table, which appeared in 1628, the logarithms were given to only ten places of decimals.

Vlacq's table was later to found to contain 603 errors, but "this cannot be regarded as a great number, when it is considered that the table was the result of an original calculation, and that more than 2,100,000 printed figures are liable to error." (Athenaeum, 15 June 1872. See also the Monthly Notices of the Royal Astronomical Society for May 1872.) An edition of Vlacq's work, containing many corrections, was issued at Leipzig in 1794 under the title Thesaurus Logarithmorum Completus by Jurij Vega.

Callet's seven-place table (Paris, 1795), instead of stopping at 100,000, gave the eight-place logarithms of the numbers between 100,000 and 108,000, in order to diminish the errors of interpolation, which were greatest in the early part of the table; and this addition was generally included in seven-place tables. The only important published extension of Vlacq's table
was made by Mr. Sang 1871, whose table contained the seven-place logarithms of all numbers below 200,000.

Briggs and Vlacq also published original tables of the logarithms of the trigonometric functions.

Besides the tables mentioned above, a great collection, called Tables du Cadastre, was constructed under the direction of Prony, by an original computation, under the auspices of the French republican government of the 1700s. This work, which contained the logarithms of all numbers up to 100,000 to nineteen places, and of the numbers between 100,000 and 200,000 to twenty-four places, exists only in manuscript, "in seventeen enormous folios," at the Observatory of Paris. It was begun in 1792; and "the whole of the calculations, which to secure greater accuracy were performed in duplicate, and the two manuscripts subsequently collated with care, were completed in the short space of two years." (English Cyclopaedia, Biography, Vol. IV., article "Prony.") Cubic interpolation could be used to find the logarithm of any number to a similar accuracy.

To the modern student who has the benefit of a calculator, the work put into the tables just mentioned is a small indication of the importance of logarithms.