Intermediate Value Theorem

Let \( P(x) \) be a polynomial with real coefficients.
Suppose that \( a \neq b \) and Suppose that \( P(a) \) and \( P(b) \) are of opposite signs (i.e. one is positive, one is negative).
Then the function \( P(x) \) must have at least one Real Zero between \( a \) and \( b \).

Let \( f(x) = -x^4 - x + 5 \)
Let \( a = 1 \) and \( b = 2 \)
\( f(a) = f(1) = 3 \) is positive, and
\( f(b) = f(2) = -13 \) is negative.

Then, by the Intermediate Value Theorem, the function \( f(x) \) must have at least one real zero between \( x = 1 \) and \( x = 2 \).

Here, approximating the \( x \)-intercept of the graph using a graphing calculator, the function \( f(x) \) has a real zero at \( x \approx 1.3794 \), where \( 1 < 1.3794 < 2 \).