Student Success Center

College Level Math Study Guide for the
ACCUPLACER (CPT)

The following sample questions are similar to the format and content of questions on the Accuplacer College Level Math test. Reviewing these samples will give you a good idea of how the test works and just what mathematical topics you may wish to review before taking the test itself. Our purposes in providing you with this information are to aid your memory and to help you do your best.

I. Factoring and expanding polynomials

Factor the following polynomials:

1. $15a^3b^2 - 45a^2b^3 - 60a^2b$
2. $7x^3y^3 + 21x^2y^2 - 10x^3y^2 - 30x^2y$
3. $6x^4y^4 - 6x^3y^2 + 8xy^2 - 8$
4. $2x^2 - 7xy + 6y^2$
5. $y^4 + y^2 - 6$
6. $7x^3 + 56y^3$
7. $81r^4 - 16s^4$
8. $(x+y)^2 + 2(x+y) + 1$

Expand the following:

9. $(x+1)(x-1)(x-3)$
10. $(2x+3y)^2$
11. $(\sqrt{3}x + \sqrt{3})(\sqrt{6}x - \sqrt{6})$
12. $(x^2 - 2x + 3)^2$
13. $(x+1)^5$
14. $(x-1)^6$

II. Simplification of Rational Algebraic Expressions

Simplify the following. Assume all variables are larger than zero.

1. $3^2 + 5 - \sqrt{4} + 4^0$
2. $9 \div 3 \cdot 5 - 8 \div 2 + 27$
3. $\sqrt{\dfrac{81}{x^4}}$
4. $2\sqrt{18} - 5\sqrt{32} + 7\sqrt{162}$
5. $\dfrac{6x-18}{3x^2+2x-8} \cdot \dfrac{12x-16}{4x-12}$

III. Solving Equations

A. Linear

1. $3 - 2(x-1) = x - 10$
2. $\dfrac{x}{2} - \dfrac{x}{7} = 1$
3. $y(y+2) = y^2 - 6$
4. $2[x - (1-3x)] = 3(x+1)$

B. Quadratic & Polynomial

1. $\left(y - \dfrac{8}{3}\right)\left(y + \dfrac{2}{3}\right) = 0$
2. $2x^3 - 4x^2 - 30x = 0$
3. $27x^3 = 1$
4. $(x-3)(x+6) = 9x + 22$
5. $t^2 + t + 1 = 0$
6. $3x^3 = 24$
7. $(x+1)^2 + x^2 = 25$
8. $5y^2 - y = 1$

C. Rational

1. $\frac{1}{y-1}+\frac{2}{y+1}=0$
2. $\frac{2}{x-3}-\frac{3}{x+3}=\frac{12}{x^2-9}$
3. $\frac{1}{6-x}+\frac{2}{x+3}=\frac{5x}{x^2-3x-18}$
4. $\frac{11}{x^2-25}-\frac{2}{x-5}=\frac{1}{x+5}$
5. $\frac{1}{a}=\frac{-6}{a^2+5}$
6. $\frac{-1}{x^2-3x}=\frac{1}{x}+\frac{x}{x-3}$

D. Absolute value

1. $|5-2z|-1=8$
2. $|x+5|-7=-2$
3. $|5x-1|=-2$
4. $\left|\frac{1}{2}x-\frac{3}{4}\right|=\frac{1}{4}$
5. $|y-1|=|7+y|$

E. Exponential

1. $10^x=1000$
2. $10^{3x+5}=100$
3. $2^{x+1}=\frac{1}{8}$
4. $3^{x^2}\left(9^x\right)=\frac{1}{3}$
5. $2^{x^2}\left(4^{2x}\right)=\frac{1}{8}$

F. Logarithmic

1. $\log_2(x+5)=\log_2(1-5x)$
2. $2\log_3(x+1)=\log_3(4x)$
3. $\log_2(x+1)+\log_2(x-1)=3$
4. $\ln x+\ln(2x+1)=0$
5. $\ln x+\ln(x+2)=\ln 3$
6. $3^{2x}=4^{x+1}$

G. Radicals

1. $4\sqrt{2y-1}-2=0$
2. $\sqrt{2x+1}+5=8$
3. $\sqrt{5x-1}-2\sqrt{x+1}=0$
4. $\sqrt{x^2+9}+x+1=0$
5. $\sqrt[3]{3x+2}+4=6$
6. $\sqrt[4]{w^2+7}=2$

IV. Solving Inequalities

Solve the following inequalities and express the answer graphically and using interval notation.

A. Linear

1. $\frac{3}{5}x+4\le -2$
2. $3(x+3)\ge 5(x-1)$
3. $3(x+2)-6>-2(x-3)+14$
4. $2\le 3x-10\le 5$

B. Absolute value: Solve and Graph.

1. $|4x+1| \le 6$
2. $|4x+3|+2 > 9$
3. $\left|\frac{x+5}{3}\right| \ge 5$
4. $|5-2x| < 15$

C. Quadratic or Rational

1. $3x^2 - 11x - 4 < 0$
2. $6x^2 + 5x \ge 4$
3. $\frac{x+2}{3-x} \ge 0$
4. $\frac{(x+1)(x-3)}{2x+7} \le 0$

V. Lines & Regions

1. Find the x and y-intercepts, the slope, and graph 6x + 5y = 30.
2. Find the x and y-intercepts, the slope, and graph x = 3.
3. Find the x and y-intercepts, the slope, and graph y = -4.
4. Write in slope-intercept form the line that passes through the points (4, 6) and (-4, 2).
5. Write in slope-intercept form the line perpendicular to the graph of 4x – y = -1 and containing the point (2, 3).
6. Graph the solution set of $x - y \ge 2$.
7. Graph the solution set of $-x + 3y < -6$.

VI. Graphing Relations, Domain & Range

For each relation, state if it is a function, state the domain & range, and graph it.

1. $y = \sqrt{x+2}$
2. $y = \sqrt{x} - 2$
3. $y = \frac{x-1}{x+2}$
4. $f(x) = -|x+1| + 3$
5. $f(x) = \frac{2x-5}{x^2-9}$
6. $x = y^2 + 2$
7. $y = x^2 + 8x - 6$
8. $y = \sqrt{-x}$
9. $y = 3^x$
10. $h(x) = \frac{6x^2}{3x^2 - 2x - 1}$

VII. Exponents and Radicals

Simplify. Assume all variables are >0. Rationalize the denominators when needed.

1. $\sqrt[3]{-8x^3}$
2. $5\sqrt{147} - 4\sqrt{48}$
3. $\sqrt{5}\left(\sqrt{15} - \sqrt{3}\right)$
4. $\left(\frac{x^{\frac{2}{3}} y^{-\frac{4}{3}}}{x^{-\frac{5}{3}}}\right)^3$
5. $\sqrt[3]{\frac{40x^4}{y^9}}$
6. $\left(\frac{54a^{-6}b^2}{9a^{-3}b^8}\right)^{-2}$
7. $\frac{\sqrt[3]{27a^3}}{\sqrt[3]{2a^2b^2}}$
8. $\frac{2}{\sqrt{5} - \sqrt{3}}$
9. $\frac{x}{\sqrt{x} + 3}$

VIII. Complex Numbers

Perform the indicated operation and simplify.

1. $\sqrt{-16} - 4\sqrt{-9}$
2. $\sqrt{-16} \cdot \sqrt{-9}$
3. $\dfrac{\sqrt{-16}}{\sqrt{-9}}$
4. $(4-3i)(4+3i)$
5. $(4-3i)^2$
6. i^{25}
7. $\dfrac{3-2i}{4+5i}$

IX. Exponential Functions and Logarithms

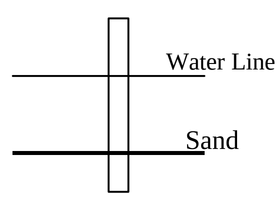
1. Graph: $f(x) = 3^x + 1$
2. Graph: $g(x) = 2^{x-1}$
3. Express $8^{-2} = \dfrac{1}{64}$ in logarithmic form.
4. Express $\log_5 25 = 2$ in exponential form.
5. Solve: $\log_2 x = 4$
6. Solve: $\log_x 9 = 2$
7. Graph: $h(x) = \log_3 x$
8. Use the properties of logarithms to expand as much as possible: $\log_4 \dfrac{3}{y}$
9. How long will it take \$850 to be worth \$1,000 if it is invested at 12% interest compounded quarterly?

X. Systems of Equations & Matrices

1. Solve the system: $\begin{aligned} 2x + 3y &= 7 \\ 6x - y &= 1 \end{aligned}$
2. Solve the system: $\begin{aligned} x + 2y + 2z &= 3 \\ 2x + 3y + 6z &= 2 \\ -x + y + z &= 0 \end{aligned}$
3. Perform the indicated operation:
   $-2\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 3\begin{bmatrix} \frac{1}{3} & -2 \\ 1 & 6 \end{bmatrix}$
4. Multiply: $\begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & 0 \\ -2 & 1 & -3 \end{bmatrix}\begin{bmatrix} 0 & 2 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
5. Find the determinant: $\begin{vmatrix} 1 & -2 \\ 3 & -1 \end{vmatrix}$
6. Find the Inverse: $\begin{bmatrix} 1 & 2 \\ -1 & 2 \end{bmatrix}$

XI. Story Problems

1. Sam made \$10 more than twice what Pete earned in one month. If together they earned \$760, how much did each earn that month?
2. A woman burns up three times as many calories running as she does when walking the same distance. If she runs 2 miles and walks 5 miles to burn up a total of 770 calories, how many calories does she burn up while running 1 mile?
3. A pole is standing in a small lake. If one-sixth of the length of the pole is in the sand at the bottom of the lake, 25 ft are in the water, and two-thirds of the total length is in the air above the water, what is the length of the pole?



XII. Conic Sections

1. Graph the following, and find the center, foci, and asymptotes if possible.

a) $(x-2)^2 + y^2 = 16$

b) $\frac{(x+1)^2}{16} + \frac{(y-2)^2}{9} = 1$

c) $\frac{(x+1)^2}{16} - \frac{(y-2)^2}{9} = 1$

d) $(x-2)^2 + y = 4$

2. Identify the conic section and put into standard form.

a) $x^2 - 4x - 12 + y^2 = 0$

b) $9x^2 + 18x + 16y^2 - 64y = 71$

c) $9x^2 + 18x - 16y^2 + 64y = 199$

d) $x^2 + y - 4x = 0$

XIII. Sequence & Series

1. Write out the first four terms of the sequence whose general term is $a_n = 3n - 2$.
2. Write out the first four terms of the sequence whose general term is $a_n = n^2 - 1$.
3. Write out the first four terms of the sequence whose general term is $a_n = 2^n + 1$.
4. Find the general term for the following sequence: 2, 5, 8, 11, 14, 17, . . .
5. Find the general term for the following sequence: 4, 2, 1, $\frac{1}{2}, \frac{1}{4}, \ldots$
6. Find the sum: $\sum_{k=0}^{6} 2k - 1$
7. Expand the following: $\sum_{k=0}^{4} \binom{4}{k} x^k y^{4-k}$

XIV. Functions

Let $f(x) = 2x + 9$ and $g(x) = 16 - x^2$. Find the following.

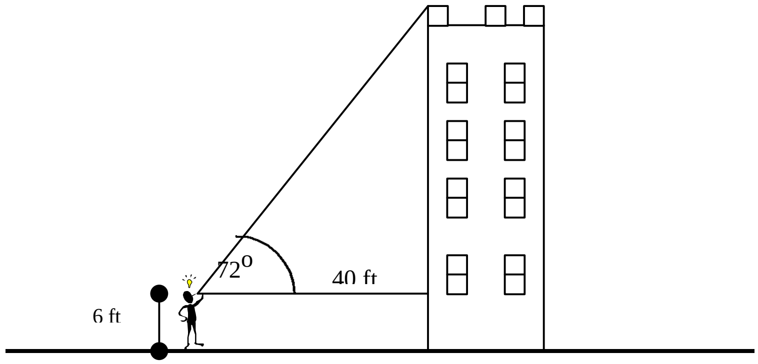
1. $f(-3) + g(2)$
2. $f(5) - g(4)$
3. $f(-1) \cdot g(-2)$
4. $f(5)/g(5)$
5. $(g \circ f)(-2)$
6. $f(g(x))$
7. $f^{-1}(2)$
8. $f(f^{-1}(3))$

XV. Fundamental Counting Rule, Factorials, Permutations, & Combinations

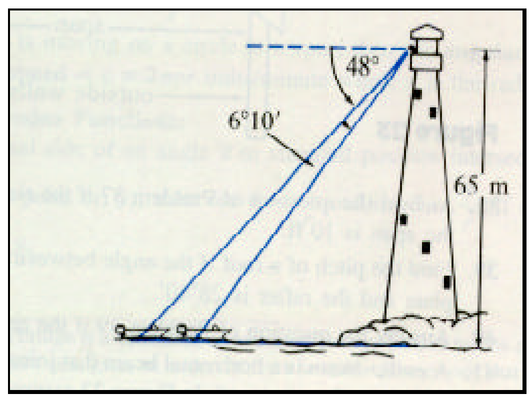
1. Evaluate: $\frac{8!}{3!(8-3)!}$
2. A particular new car model is available with five choices of color, three choices of transmission, four types of interior, and two types of engines. How many different variations of this model care are possible?
3. In a horse race, how many different finishes among the first three places are possible for a ten-horse race?
4. How many ways can a three-person subcommittee be selected from a committee of seven people? How many ways can a president, vice president, and secretary be chosen from a committee of seven people.

XVI. Trigonometry

1. Graph the following through one period: $f(x) = \sin x$
2. Graph the following through one period: $g(x) = \cos(2x)$
3. A man whose eye level is 6 feet above the ground stands 40 feet from a building. The angle of elevation from eye level to the top of the building is 72°. How tall is the building.



4. 4. A man standing at the top of a 65m lighthouse observes two boats. Using the data given in the picture, determine the distance between the two boats.



Answers

I. Factoring and Expanding Polynomials

When factoring, there are three steps to keep in mind.
1. Always factor out the Greatest Common Factor
2. Factor what is left
3. If there are four terms, consider factoring by grouping.

Answers:

1. $15a^2b(ab-3b^2-4)$
2. $7x^3y^3+21x^2y^2-10x^3y^2-30x^2y$

   $x^2y(7xy^2+21y-10xy-30)$
   $x^2y[(7xy^2+21y)+(-10xy-30)]$
   $x^2y[7y(xy+3)-10(xy+3)]$
   $x^2y(7y-10)(xy+3)$

   Since there are 4 terms, we consider factoring by grouping. First, take out the Greatest Common Factor.

   When you factor by grouping, be careful of the minus sign between the two middle terms.

3. $2(3x^3y^2+4)(xy^2-1)$
4. $(2x-3y)(x-2y)$
5. y^4+y^2-6

   u^2+u-6
   $(u-2)(u+3)$
   $(y^2-2)(y^2+3)$

   When a problem looks slightly odd, we can make it appear more natural to us by using substitution (a procedure needed for calculus). Let $u=y^2$. Factor the expression with u's. Then, substitute the y^2 back in place of the u's. If you can factor more, proceed. Otherwise, you are done.

6. $7(x+2y)(x^2-2xy+4y^2)$

   Formula for factoring the sum of two cubes:
   $a^3+b^3=(a+b)(a^2-ab+b^2)$
   The difference of two cubes is:
   $a^3-b^3=(a-b)(a^2+ab+b^2)$

7. $(3r-2s)(3r+2s)(9r^2+4s^2)$
8. $(x+y+1)^2$ Hint: Let u = x + y
9. x^3-3x^2-x+3
10. $4x^2+12xy+9y^2$
11. $3\sqrt{2}x^2-3\sqrt{2}$
12. $x^4-4x^3+10x^2-12x+9$
13. $x^5+5x^4+10x^3+10x^2+5x+1$
14. $x^6-6x^5+15x^4-20x^3+15x^2-6x+1$

When doing problems 13 and 14, you may want to use Pascal's Triangle.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

II. Simplification of Rational Algebraic Expressions

1. 13
2. 38
3. $\frac{9}{x^2}$
4. $49\sqrt{2}$
5. $\frac{6}{x+2}$

If you have $\sqrt{4}$, you can write 4 as a product of primes ($2 \cdot 2$). In square roots, it takes two of the same thing on the inside to get one thing on the outside: $\sqrt{4} = \sqrt{2 \cdot 2} = 2$.

III. Solving Equations

A. Linear

1. x = 5 2. $x = \frac{14}{5}$ or $2\frac{4}{5}$ 3. y = -3 4. x = 1

B. Quadratic & Polynomials

1. $y = \frac{8}{3}, -\frac{2}{3}$
2. x = 0, -3, 5
3. $x = \frac{1}{3}, \frac{-1 \pm i\sqrt{3}}{6}$
4. x = 10, -4
5. $t = \frac{-1 \pm i\sqrt{3}}{2}$
6. $x = 2, -1 \pm i\sqrt{3}$
7. x = 3, -4
8. $y = \frac{1 \pm \sqrt{21}}{10}$

Solving quadratics or Polynomials:
1. Try to factor
2. If factoring is not possible, use the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ where } ax^2 + bx + c = 0.$$

Note: $i = \sqrt{-1}$ and that $\sqrt{-12} = i\sqrt{12} = i\sqrt{2 \cdot 2 \cdot 3} = 2i\sqrt{3}$

C. Rational

1.
$$\frac{1}{y-1} + \frac{2}{y+1} = 0$$
$$(y-1)(y+1)\left[\frac{1}{y-1} + \frac{2}{y+1}\right] = 0(y-1)(y+1)$$
$$(y-1)(y+1)\frac{1}{y-1} + (y-1)(y+1)\frac{2}{y+1} = 0$$
$$(y+1) + 2(y-1) = 0$$
$$3y - 1 = 0$$
$$y = \frac{1}{3}$$

Solving Rational Equations:
1. Find the lowest common denominator for all fractions in the equation
2. Multiply both sides of the equation by the lowest common denominator
3. Simplify and solve for the given variable
4. Check answers to make sure that they do not cause zero to occur in the denominators of the original equation

2. Working the problem, we get x = 3. However, 3 causes the denominators to be zero in the original equation. Hence, this problem has no solution.

3. $x = -\frac{15}{4}$
4. x = 2
5. a = -1, -5
6. x = -2, 1

D. Absolute Value

1. $|5-2z|-1=8$

$$|5-2z|=9$$
$$5-2z=9 \text{ or } 5-2z=-9$$
$$-2z=4 \text{ or } -2z=-14$$
$$z=-2 \text{ or } z=7$$

2. x = 0 or –10
3. No solution! An absolute value can not equal a negative number.
4. x = 2 or 1
5. $|y-1|=|7+y|$

$$y-1=7+y \text{ or } y-1=-(7+y)$$
$$0=8 \quad \text{or} \quad 2y=-6$$
$$\text{No Solution} \quad \text{or} \quad y=-3$$
$$\text{Hence, } y=-3 \text{ is the only solution.}$$

Solving Absolute Value Equations:

1. Isolate the Absolute value on one side of the equation and everything else on the other side.
2. Remember that $|x|=2$ means that the object inside the absolute value has a distance of 2 away from zero. The only numbers with a distance of 2 away from zero are 2 and –2. Hence, x = 2 or x = -2. Use the same thought process for solving other absolute value equations.

Note: An absolute value can **not** equal a negative value. $|x|=-2$ does not make any sense.

Note: Always check your answers!!

E. Exponential

1. $10^x = 1000$

$$10^x = 10^3$$
$$\therefore x = 3$$

2. x = -1
3. x = -4
4. x = -1, -1
5. x = -1, -3

Some properties you will need to be familiar with.

If $a^r = a^s$, then r = s.

If $a^r = b^r$, then a = b.

F. Logarithms

1. $\log_2(x+5) = \log_2(1-5x)$

$$x+5 = 1-5x$$
$$6x = -4$$
$$x = -\frac{2}{3}$$

2. $2\log_3(x+1) = \log_3(4x)$

$$\log_3(x+1)^2 = \log_3(4x)$$
$$(x+1)^2 = 4x$$
$$x^2 - 2x + 1 = 0$$
$$x = 1, 1$$

3. x = 3 is the only solution since –3 cause the argument of a logarithm to be negative.

Properties of logarithms to be familiar with.

- If $\log_b M = \log_b N$, then M = N.
- If $\log_b x = y$, then this equation can be rewritten in exponential form as $b^y = x$.
- $\log_b(M \cdot N) = \log_b M + \log_b N$
- $\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$
- $\log_b M^r = r \cdot \log_b M$
- Always check your answer!! Bases and arguments of logarithms can not be negative.

4. $x = \frac{1}{2}$ is the only solution since –1 causes the argument of a logarithm to be negative.

5. $x = 1$ is the only solution since –3 causes the argument of a logarithm to be negative.

6.
$$3^{2x} = 4^{x+1}$$
$$2x \ln 3 = (x+1)\ln 4$$
$$2x \ln 3 - x \ln 4 = \ln 4$$
$$x(2\ln 3 - \ln 4) = \ln 4$$
$$x = \frac{\ln 4}{2\ln 3 - \ln 4}$$

G. Radicals

1.
$$4\sqrt{2y-1} - 2 = 0$$
$$\sqrt{2y-1} = \frac{1}{2}$$
$$2y - 1 = \frac{1}{4}$$
$$2y = \frac{5}{4}$$
$$y = \frac{5}{8}$$

2. x = 4

3.
$$\sqrt{5x-1} - 2\sqrt{x+1} = 0$$
$$\sqrt{5x-1} = 2\sqrt{x+1}$$
$$\left(\sqrt{5x-1}\right)^2 = \left(2\sqrt{x+1}\right)^2$$
$$5x - 1 = 4(x+1)$$
$$x = -3$$

Checking your answer you find that $4i - 2i\sqrt{2} = 0$. Since this is not possible, there is no solution for this problem.

4. No solution. x = 4 does not work in the original equation.
5. x = 2
6. w = 3, -3

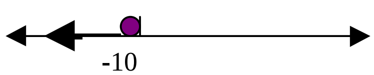
Solving Equations with radicals:
1. Isolate the radical on one side of the equation and everything else on the other side.
2. If it is a square root, then square both sides. If it is a cube root, then cube both sides, etc…
3. Solve for the given variable and check your answer.

Note: A radical with an even index such as $\sqrt{\ }, \sqrt[4]{\ }, \sqrt[6]{\ }, \ldots$ can **not** have a negative argument (The square root can but you must use complex numbers).

IV. Solving Inequalities

A. Linear

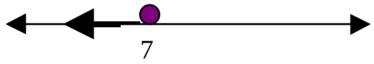
1.
$$\frac{3}{5}x + 4 \le -2$$
$$\frac{3}{5}x \le -6$$
$$x \le -10$$



Interval Notation: $(-\infty, -10]$

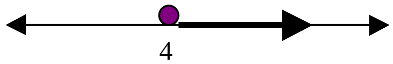
When solving linear inequalities, you use the same steps as solving an equation. The difference is when you multiply or divide both sides by a negative number, you must change the direction of the inequality.
For example:
$$5 > 3$$
$$-1(5) < -1(3)$$
$$-5 < -3$$

2. $x \le 7$



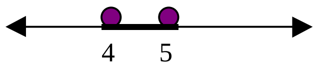
Interval Notation: $(-\infty, 7]$

3. $x > 4$



Interval Notation: $(4, \infty)$

4. $4 \le x \le 5$

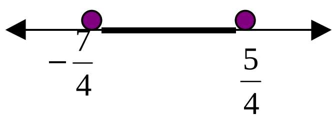


Interval Notation: $[4, 5]$

B. Absolute Value

1. $|4x + 1| \le 6$

$$-6 \le 4x + 1 \le 6$$
$$-7 \le 4x \le 5$$
$$-\frac{7}{4} \le x \le \frac{5}{4}$$



Interval: $\left[-\frac{7}{4}, \frac{5}{4}\right]$

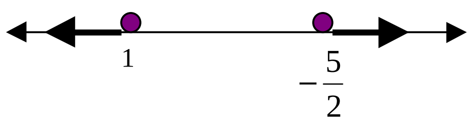
Think of the inequality sign as an alligator. If the alligator is facing away from the absolute value sign such as, $|x| < 5$, then one can remove the absolute value and write $-5 < x < 5$. This expression indicates that x can not be farther than 5 units away from zero.

If the alligator faces the absolute value such as, $|x| > 5$, then one can remove the absolute value and write $x > 5$ or $x < -5$. These expressions express that x can not be less than 5 units away from zero.

For more information, see page 132 in the College Algebra text.

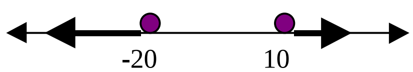
2. $x > 1$ or $x < -\frac{5}{2}$

Interval: $\left(-\infty, -\frac{5}{2}\right) \cup (1, \infty)$



3. $x \le -20$ or $x \ge 10$

Interval: $(-\infty, -20] \cup [10, \infty)$



4. $-5 < x < 10$

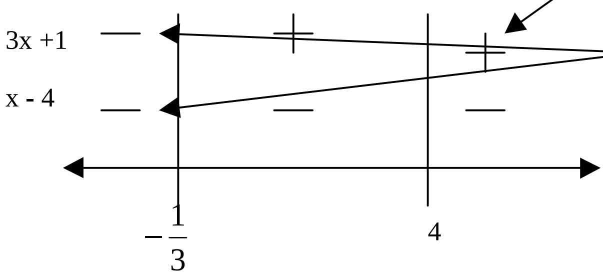
Interval: $(-5, 10)$

C. Quadratic or Rational

1. $3x^2 - 11x - 4 < 0$

   $(3x+1)(x-4) < 0$

   $x = -\frac{1}{3}$ and $x = 4$ make the above factors zero.



Answer: $\left(-\frac{1}{3}, 4\right)$

2. $\left(-\infty, -\frac{4}{3}\right] \cup \left[\frac{1}{2}, \infty\right)$
3. $[-2, 3)$
4. $\left(-\infty, -\frac{7}{2}\right) \cup [-1, 3]$

Steps to solving quadratic or rational inequalities.

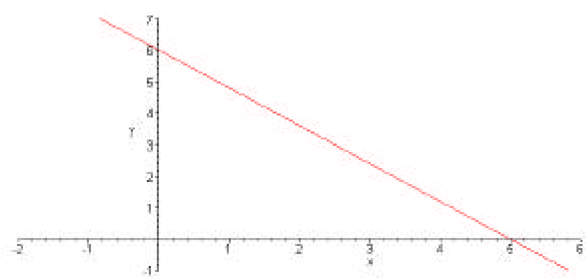
1. Zero should be on one side of the inequality while every thing else is on the other side.
2. Factor
3. Set the factors equal to zero and solve.
4. Draw a chart. You should have a number line and lines dividing regions on the numbers that make the factors zero. Write the factors in on the side.
5. In each region, pick a number and substitute it in for x in each factor. Record the sign in that region.
6. In our example, 3x+1 is negative in the first region when we substitute a number such as –2 in for x. Moreover, 3x+1 will be negative everywhere in the first region. Likewise, x-4 will be negative throughout the whole first region. If x is a number in the first region, then both factors will be negative. Since a negative times a negative number is positive, x in the first region is not a solution. Continue with step 5 until you find a region that satisfies the inequality.
7. Especially with rational expression, check that your endpoints do not make the original inequality undefined.

Page 145 of the College Algebra text discusses this topic in detail.

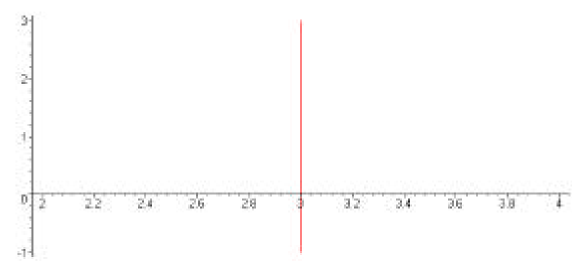
V. Lines and Regions

For details on how to solve these problems, see page 263 of the College Algebra text.

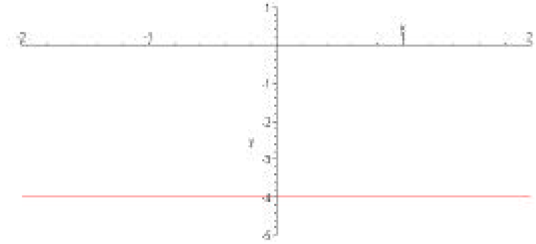
1. x – intercept: (5, 0)
   y – intercept: (0, 6)
   slope: $-\frac{6}{5}$



2. x – intercept: (3, 0)
   y – intercept: None
   slope: None



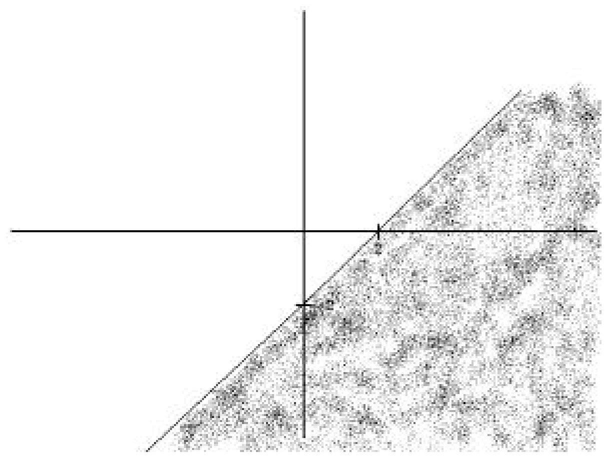
3. x – intercept: None
y – intercept: (0, -4)
slope: 0



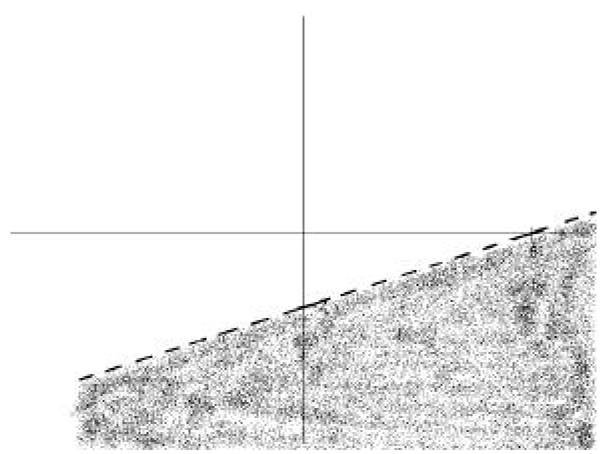
4. $y = \frac{1}{2}x + 4$

5. $y = -\frac{1}{4}x + 3\frac{1}{2}$

6. $x - y \geq 2$



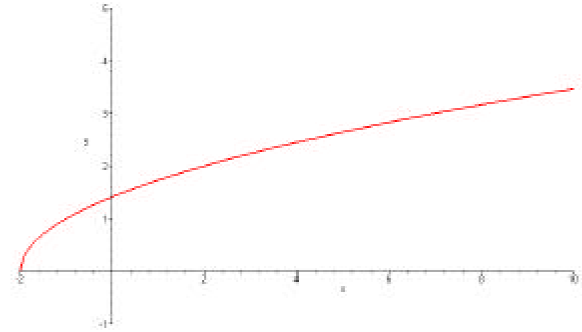
7. $-x + 3y < -6$



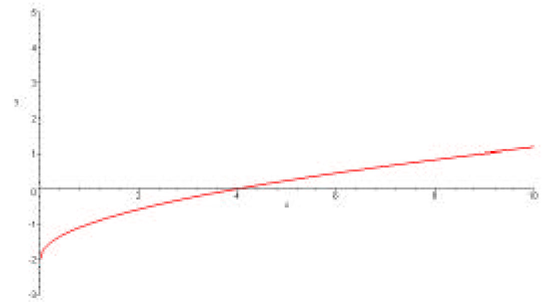
VI. Graphing Relations

For details on how to solve these problems, see Chapter 2 of the College Algebra text.

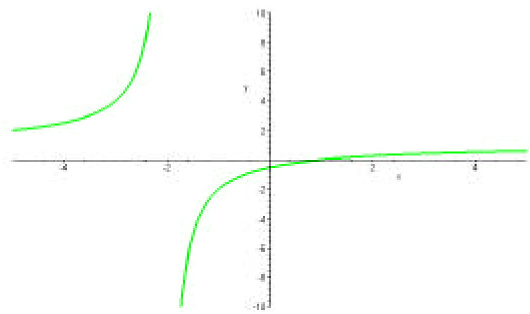
1. $y = \sqrt{x+2}$
   Domain: $[-2, \infty)$
   Range: $[0, \infty)$



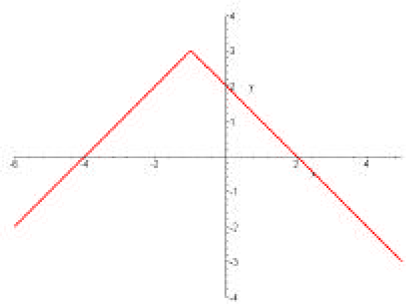
2. $y = \sqrt{x} - 2$
   Domain: $[0, \infty)$
   Range: $[-2, \infty)$



3. $y = \frac{x-1}{x+2}$
   Domain: All Real Numbers except -2
   Range: $(-\infty, 1) \cup (1, \infty)$



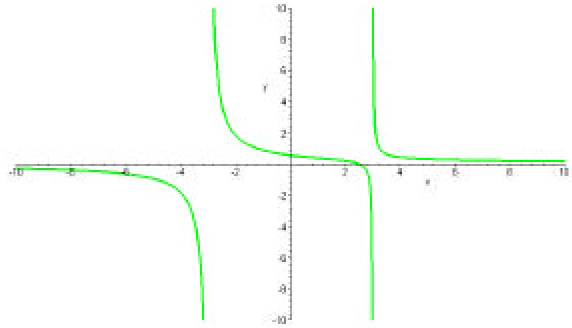
4. $f(x) = -|x+1| + 3$
   Domain: $(-\infty, \infty)$
   Range: $(-\infty, 3]$



5. $f(x) = \dfrac{2x-5}{x^2-9}$

Domain: All Real Numbers except ± 3

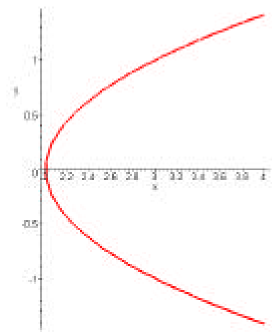
Range: All Real Numbers.



6. $x = y^2 + 2$

Domain: $[2, \infty)$

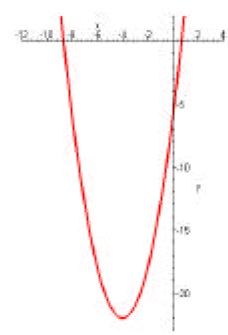
Range: $(-\infty, \infty)$



7. $y = x^2 + 8x - 6$

Domain: $(-\infty, \infty)$

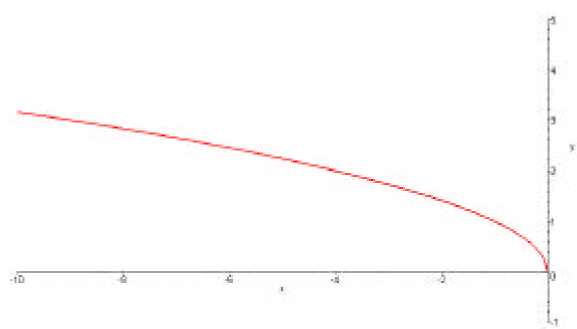
Range: $[-6, \infty)$



7. $y = \sqrt{-x}$

Domain: $(-\infty, 0]$

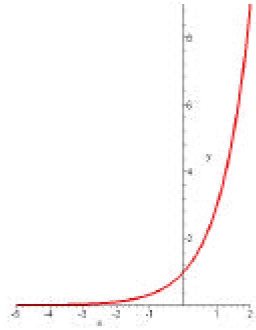
Range: $[0, \infty)$



8. $y = 3^x$

Domain: $(-\infty, \infty)$

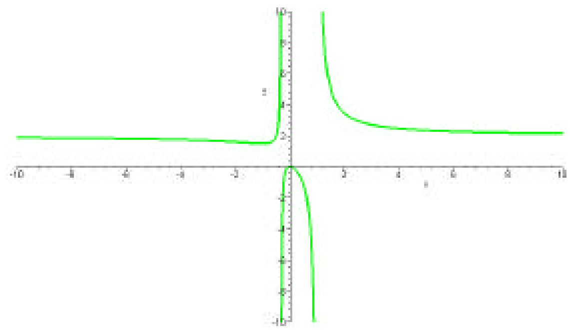
Range: $(0, \infty)$



9. $h(x) = \frac{6x^2}{3x^2 - 2x - 1}$

Domain: All Real Numbers except $-\frac{1}{3}, 1$

Range: $(2, \infty) \cup (-\infty, 0]$



VII. Exponents and Radicals

For details on how to solve these problems, see page 29 and 361 of the College Algebra text.

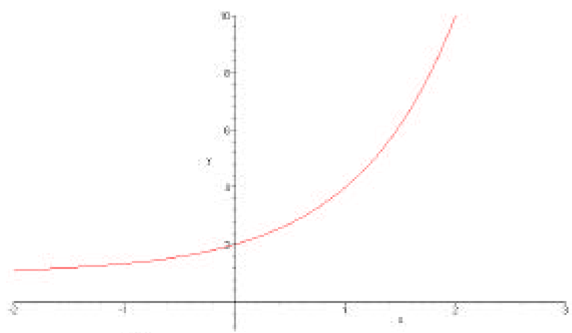
1. $-2x$
2. $5\sqrt{147} - 4\sqrt{48} = 35\sqrt{3} - 16\sqrt{3} = 19\sqrt{3}$
3. $5\sqrt{3} - \sqrt{15}$
4. $\frac{x^7}{y^4}$
5. $\frac{2x\sqrt{5x}}{y^3}$
6. $\left(\frac{54a^{-6}b^2}{9a^{-3}b^8}\right)^{-2} = \left(\frac{6}{a^3b^6}\right)^{-2} = \frac{a^6b^{12}}{36}$
7. $\frac{\sqrt[3]{27a^3}}{\sqrt[3]{2a^2b^2}} = \frac{3a}{\sqrt[3]{2a^2b^2}} = \frac{3a}{\sqrt[3]{2a^2b^2}} \cdot \frac{\sqrt[3]{4ab}}{\sqrt[3]{4ab}} = \frac{3a\sqrt[3]{4ab}}{2ab} = \frac{3\sqrt[3]{4ab}}{2b}$
8. $\sqrt{5} + \sqrt{3}$
9. $\left(\frac{x}{\sqrt{x}+3}\right)\left(\frac{\sqrt{x}-3}{\sqrt{x}-3}\right) = \frac{x\sqrt{x} - 3x}{x - 9}$

VIII. Complex Numbers

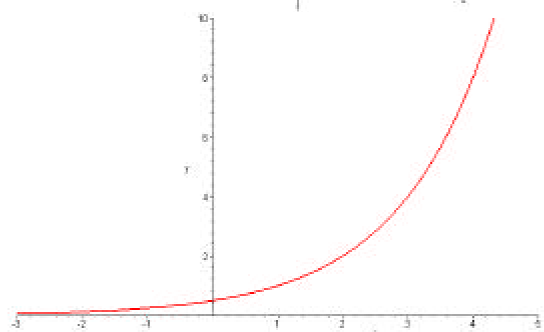
1. $\sqrt{-16} - 4\sqrt{-9} = 4i - 12i = -8i$
2. $\sqrt{-16} \cdot \sqrt{-9} = (4i)(3i) = 12i^2 = -12$
3. $\dfrac{\sqrt{-16}}{\sqrt{-9}} = \dfrac{4i}{3i} = \dfrac{4i}{3i} \cdot \dfrac{3i}{3i} = \dfrac{12i^2}{9i^2} = \dfrac{-12}{-9} = \dfrac{4}{3}$
4. $(4-3i)(4+3i) = 16 - 9i^2 = 16 + 9 = 25$
5. $(4-3i)^2 = (4-3i)(4-3i) = 16 - 24i + 9i^2 = 16 - 24i - 9 = 7 - 24i$
6. $i^{25} = i \cdot i^{24} = i\left(i^2\right)^{12} = i(-1)^{12} = i$
7. $\dfrac{3-2i}{4+5i} \cdot \dfrac{4-5i}{4-5i} = \dfrac{12 - 19i + 10i^2}{16 - 25i^2} = \dfrac{12 - 19i - 10}{16 + 25} = \dfrac{2 - 19i}{41}$

IX. Exponential Functions and Logarithms

1. $f(x) = 3^x + 1$



2. $g(x) = 2^{x-1}$



3. $\log_8 \dfrac{1}{64} = -2$

4. $5^2 = 25$

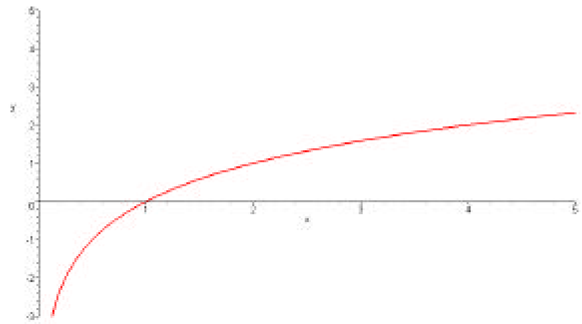
5. $\log_2 x = 4$

$2^4 = x$

$16 = x$

6. x = 3; -3 is not a solution because bases are not allowed to be negative

7. $h(x) = \log_3 x$



8. $\log_4 \frac{3}{y} = \log_4 3 - \log_4 y$

9. $A = P\left(1 + \frac{r}{n}\right)^{nt}$ where $\begin{cases} A = \text{money ended with} \\ P = \text{Principle started with} \\ r = \text{yearly interest rate} \\ n = \text{number of compounds per year} \\ t = \text{number of years} \end{cases}$

$$1000 = 850\left(1 + \frac{.12}{4}\right)^{4t}$$

$$\frac{20}{17} = \left(1 + \frac{.12}{4}\right)^{4t}$$

$$\log\left(\frac{20}{17}\right) = \log\left(1 + \frac{.12}{4}\right)^{4t}$$

$$\log\left(\frac{20}{17}\right) = 4t\log\left(1 + \frac{.12}{4}\right)$$

$$\frac{\log\left(\frac{20}{17}\right)}{4\log\left(1 + \frac{.12}{4}\right)} = t$$

X. Systems of equations

Please see chapter 5 of the College Algebra text for an explanation of solving linear systems and

1. $\left(- \quad \right)$

2. $\begin{bmatrix} -\frac{}{5}k+\frac{}{5} \\ -\frac{8}{}k+\frac{2}{} \\ k \end{bmatrix}$ $k \in$ numbers

3. $\begin{bmatrix} -5 & -8 \\ 5 & 14 \end{bmatrix}$

4. $\begin{bmatrix} -1 & 0 & 2 \\ 2 & 4 & 0 \\ 1 & -2 & -5 \end{bmatrix}$

5. 4

6. $\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} \end{bmatrix}$

XI. Story Problems

1. Let x = the money Pete earns
   2x+10=the money Sam earns
   $(2x+10)+x=760$
   Pete earns $250
   Sam earns $510

2. x = burned calories walking
   3x = burned calories running
   $2(3x)+5x=770$
   x = 70
   Answer: 210 Calories

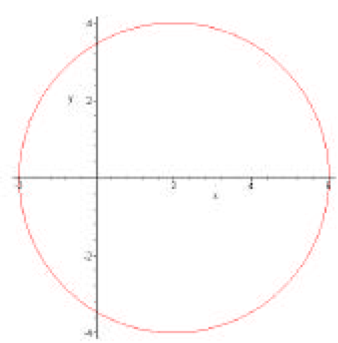
3. x = length of pole
   $\frac{2}{3}x+25+\frac{1}{6}x=x$
   Answer: 150 feet

XII. Conic Sections

For an explanation of the theory behind the following problems, see chapter 7 of the College Algebra text.

1. a) $(x-2)^2+y^2=16$

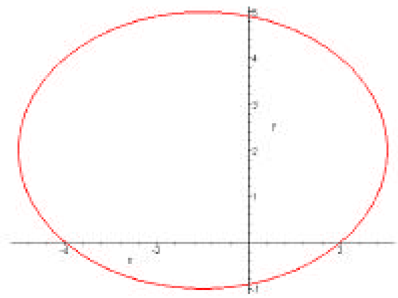
   Center: (2, 0)
   Radius: 4



b) $\dfrac{(x+1)^2}{16}+\dfrac{(y-2)^2}{9}=1$

Center: $(-1,2)$

Foci: $\left(-1\pm\sqrt{7},2\right)$



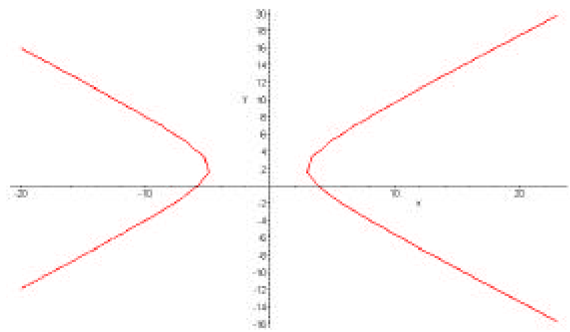
c) $\dfrac{(x+1)^2}{16}-\dfrac{(y-2)^2}{9}=1$

Center: $(-1,2)$

Foci: $(-6,2),(4,2)$

Asymptotes: $y=\dfrac{3}{4}x-\dfrac{3}{4}$

$y=-\dfrac{3}{4}x+\dfrac{5}{4}$

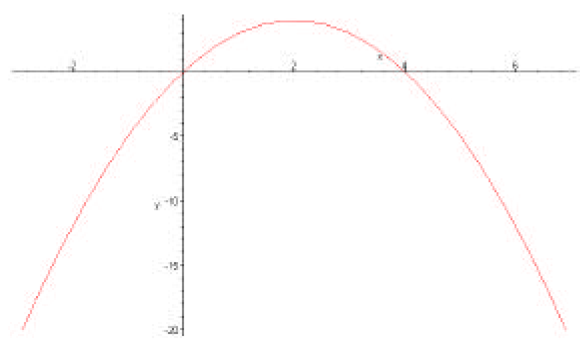


d) $(x-2)^2+y=4$

Vertex: $(2,4)$

Foci: $\left(2,\dfrac{15}{4}\right)$

Directrix: $y=\dfrac{17}{4}$



2. a) Circle $(x-2)^2+y^2=16$

b) Ellipse $\dfrac{(x+1)^2}{16}+\dfrac{(y-2)^2}{9}=1$

c) Hyperbola $\dfrac{(x+1)^2}{16}-\dfrac{(y-2)^2}{9}=1$

d) Parabola $y=-(x-2)^2+4$

XIII. Sequence and Series

For an explanation of Sequences and Series, see Chapter 8 in the College Algebra text.

1. 1, 4, 7, 10
2. 0, 3, 8, 15
3. 3, 5, 9, 17
4. $a_n = 3n - 1$
5. $a_n = 4 \cdot \left(\frac{1}{2}\right)^{n-1}$
6. $\sum_{k=0}^{6}(2k-1) = -1+1+3+7+9+11 = 30$
7. $\sum_{k=0}^{4}\binom{4}{k}x^k y^{4-k} = y^4 + 4xy^3 + 6x^2y^2 + 4x^3y + x^4$

XIV. Functions

For an explanation of function notation, see page 176 in the College Algebra text.

1. $f(-3)+g(2) = 3+14 = 17$
2. $f(5)-g(4) = 19-12 = 7$
3. $f(-1)\cdot g(-2) = 7\cdot 18 = 126$
4. $\frac{f(5)}{g(5)} = \frac{19}{11}$
5. $(g \circ f)(-2) = g(f(-2)) = g(5) = 11$
6. $f(g(x)) = f(16-x) = 41-2x$
7. $f^{-1}(x) = \frac{x-9}{2}$; $f^{-1}(2) = \frac{2-9}{2} = -\frac{7}{2}$
8. 3

XV. Fundamental Counting Rule, Permutations, & Combinations

See chapter 8 for assistance with the counting rules.

1. 56

2. 120

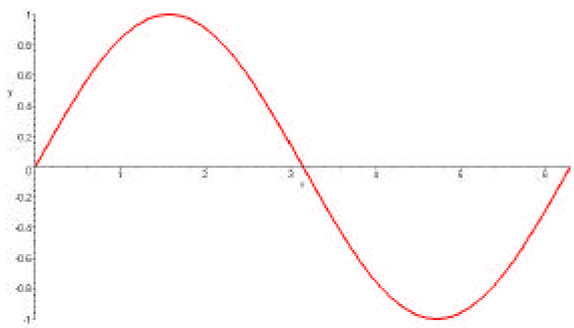
3. 720

4. Committee 35
   Elected 210

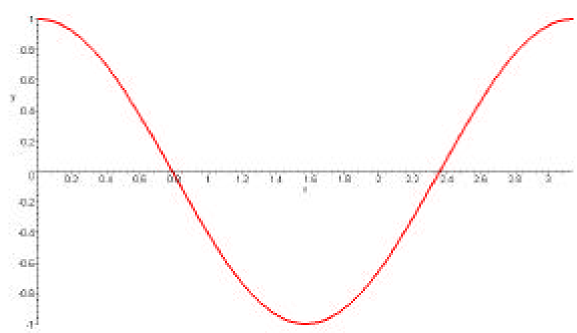
XVI. Trigonometry

For assistance, see the text, Fundamentals of Trigonometry on reserve in the Aims Community College's library.

1. $f(x) = \sin x$



2. $g(x) = \cos(2x)$



3. $x = 6 + 40\tan 72 \approx 129.1$
4. $\text{Distance between the boats} = 65\tan 42 - 65\tan 35^{\circ}50' \approx 11.59\text{meters}$