Student Success Center

Elementary Algebra Study Guide for the
ACCUPLACER (CPT)

The following sample questions are similar to the format and content of questions on the Accuplacer Elementary Algebra test. Reviewing these samples will give you a good idea of how the test works and just what mathematical topics you may wish to review before taking the test itself. Our purposes in providing you with this information are to aid your memory and to help you do your best.

I. Order of operations

1. $3^2 + 5 - \sqrt{4} + 4^0$
2. $(5+1)(4-2)-3$
3. $3 \cdot 7^2$
4. $2(7+3)^2$
5. $49 \div 7 - 2 \cdot 2$
6. $9 \div 3 \cdot 5 - 8 \div 2 + 27$
7. $3 + 2(5) - |-7|$
8. $\dfrac{5 \cdot 5 - 4(4)}{2^2 - 1}$
9. $\dfrac{4^2 - 5^2}{(4-5)^2}$
10. -5^2

II. Scientific Notation

Write the following in Scientific Notation.

1. 350,000,000
2. 0.0000000000000523
3. 120,500,000,000,000,000,000

Write in expanded form.

4. 6.02×10^{23}
5. 3.0×10^{8}
6. 1.819×10^{-9}

Simplify. Write answers in scientific notation.

7. $(3 \times 10^3)(5 \times 10^6)$
8. $(3 \times 10^{-4})^2$
9. $\dfrac{6 \times 10^9}{3 \times 10^4}$
10. $\dfrac{(3.2 \times 10^5)(2 \times 10^{-3})}{2 \times 10^{-5}}$

III. Substitution

Find each value if $x = 3$, $y = -4$, and $z = 2$.

1. $xyz - 4z$
2. $2x - y$
3. $x(y - 3z)$
4. $\dfrac{5x - z}{xy}$
5. $3y^2 - 2x + 4z$

IV. Linear equations in one variable

Solve the following for x.

1. $6x - 48 = 6$
2. $\dfrac{2}{3}x - 5 = x - 3$
3. $50 - x - (3x + 2) = 0$
4. $8 - 4(x - 1) = 2 + 3(4 - x)$

V. Formulas

1. Solve $PV = nRT$ for T.
2. Solve y = 3x + 2 for x.
3. Solve $C = 2pr$ for r.
4. Solve $\frac{x}{2} + \frac{y}{5} = 1$ for y.
5. Solve $y = hx + 4x$ for x.

VI. Word Problems

1. One number is 5 more than twice another number. The sum of the numbers is 35. Find the numbers.

2. Ms. Jones invested $18,000 in two accounts. One account pays 6% simple interest and the other pays 8%. Her total interest for the year was $1,290. How much did she have in each account?

3. How many liters of a 40% solution and an16% solution must be mixed to obtain 20 liters of a 22% solution?

4. Sheila bought burgers and fries for her children and some friends. The burgers cost $2.05 each and the fries are $.85 each. She bought a total of 14 items, for a total cost of $19.10. How many of each did she buy?

VII. Inequalities

Solve and graph on the number line.

1. $2x - 7 \geq 3$
2. $-5(2x + 3) < 2x - 3$
3. $3(x - 4) - (x + 1) \leq -12$

VIII. Exponents & polynomials

Simplify and write answers with positive exponents.

1. $(3x^2 - 5x - 6) + (5x^2 + 4x + 4)$
2. $\dfrac{(2a^{-5}b^4c^3)^{-2}}{(3a^3b^{-7}c^3)^2}$
3. $(3x^0y^5z^6)(-2xy^3z^{-2})$
4. $(-a^5b^7c^9)^4$
5. $(4x^2y^6z)^2(-x^{-2}y^3z^4)^6$
6. $\dfrac{24x^4 - 32x^3 + 16x^2}{8x^2}$
7. $(x^2 - 5x)(2x^3 - 7)$
8. $\dfrac{26a^2b^{-5}c^9}{-4a^{-6}bc^9}$
9. $(5a + 6)^2$

IX. Factoring

1. $x^2 + 5x - 6$
2. $x^2 - 5x - 6$
3. $4x^2 - 36$
4. $x^2 + 4$
5. $64x^4 - 4y^4$
6. $8x^3 - 27$
7. $49y^2 + 84y + 36$
8. $12x^2 + 12x + 3$

X. Quadratic Equations

1. $4a^2 + 9a + 2 = 0$
2. $9x^2 - 81 = 0$
3. $25x^2 - 6 = 30$
4. $3x^2 - 5x - 2 = 0$
5. $(3x + 2)^2 = 16$
6. $r^2 - 2r - 4 = 0$

XI. Rational Expressions

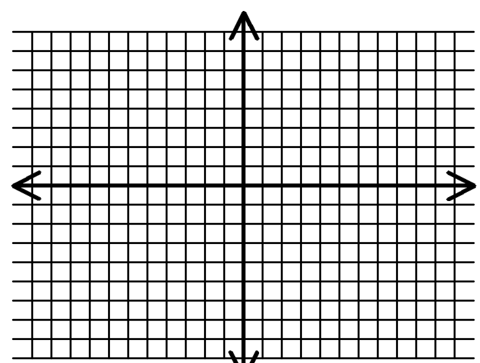
Perform the following operations and simplify where possible. If given an equation, solve for the variable.

1. $\dfrac{4}{2a-2} + \dfrac{3a}{a^2 - a}$
2. $\dfrac{3}{x^2 - 1} - \dfrac{4}{x^2 + 3x + 2}$
3. $\dfrac{6x - 18}{3x^2 + 2x - 8} \cdot \dfrac{12x - 16}{4x - 12}$
4. $\dfrac{16 - x^2}{x^2 + 2x - 8} \div \dfrac{x^2 - 2x - 8}{4 - x^2}$
5. $\dfrac{x^3 - 1}{x - 1}$
6. $\dfrac{\frac{2}{x} - \frac{1}{y}}{\frac{1}{xy}}$
7. $\dfrac{2}{x-1} + \dfrac{1}{x+1} = \dfrac{5}{4}$
8. $\dfrac{3}{k} + 1 = \dfrac{3+k}{2k}$
9. $\dfrac{5-x}{x} + \dfrac{3}{4} = \dfrac{7}{x}$

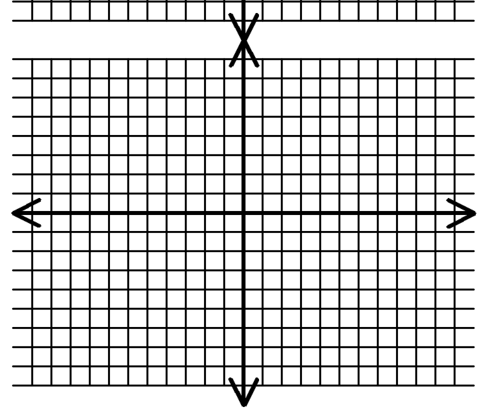
XII. Graphing

Graph each equation on the coordinate axis.

1. $3x - 2y = 6$
2. $x = -3$
3. $y = 2$
4. $y = \dfrac{-2}{3}x + 5$



5. $y = |x - 3|$
6. $y = -x^2 + 2$
7. $y = \sqrt{x + 2}$



XIII. Systems of Equations

Solve the following systems of equations.

1. $\begin{aligned} 2x - 3y &= -12 \\ x - 2y &= -9 \end{aligned}$

2. $\begin{aligned} 4x + 6y &= 10 \\ 2x + 3y &= 5 \end{aligned}$

3. $\begin{aligned} x + 2y &= 5 \\ x + 2y &= 7 \end{aligned}$

4. $\begin{aligned} 2x - 3y &= -4 \\ y &= -2x + 4 \end{aligned}$

XIV. Radicals

Simplify the following using the rules of radicals (rationalize denominators). All variables represent positive numbers.

1. $\left(\sqrt{8}\right)\left(\sqrt{10}\right)$
2. $\sqrt[4]{\frac{81}{x^4}}$
3. $\sqrt{\frac{4}{3}}$
4. $\sqrt{\frac{12}{18}} \cdot \sqrt{\frac{15}{40}}$
5. $\sqrt[3]{24x^3y^6}$
6. $2\sqrt{18} - 5\sqrt{32} + 7\sqrt{162}$
7. $\frac{\sqrt{3}}{5 - \sqrt{3}}$
8. $\left(2\sqrt{3} + 5\sqrt{2}\right)\left(3\sqrt{3} - 4\sqrt{2}\right)$

Answers

I. Order of Operations

When working with $(\ ), 2^{\text{exponents}}, \bullet, \div, -, \text{and} +$, one must remember the order of the operations. First, parenthesis or exponents as one calculates from left to right. Second, multiplication or division as one calculates from the left to right. And finally, addition or subtraction as one calculates from left to right.

1. $3^2 + 5 - \sqrt{4} + 4^0 = 9 + 5 - 2 + 1 = 14 - 2 + 1 = 12 + 1 = 13$
2. $(5+1)(4-2) - 3 = (6)(2) - 3 = 12 - 3 = 9$
3. 147
4. 200
5. 3
6. 38
7. $3 + 2(5) - |-7| = 3 + 10 - 7 = 13 - 7 = 6$
8. $\frac{5 \cdot 5 - 4(4)}{2^2 - 1} = \frac{25 - 16}{4 - 1} = \frac{9}{3} = 3$
9. -9
10. –25

II. Scientific Notation

All numbers in scientific notation have the following form: $\text{nonzerodigit.restofnumber} \times 10^{\text{power}}$.

1. $350{,}000{,}000 = 3.5 \times 10^{8}$
2. $0.0000000000000523 = 5.23 \times 10^{-14}$
3. $120{,}500{,}000{,}000{,}000{,}000{,}000 = 1.205 \times 10^{20}$
4. $602{,}000{,}000{,}000{,}000{,}000{,}000{,}000$
5. $300{,}000{,}000$
6. 0.00000001819
7. $(3\times10^{3})(5\times10^{6}) = 3\cdot5\times10^{3}\cdot10^{6} = 15\times10^{9} = 1.5\times10^{10}$
8. $(3\times10^{-4})^{2} = 3^{2}\times(10^{-4})^{2} = 9\times10^{-8}$
9. $\dfrac{6\times10^{9}}{3\times10^{4}} = \dfrac{6}{3}\times\dfrac{10^{9}}{10^{4}} = 2\times10^{5}$
10. $\dfrac{(3.2\times10^{5})(2\times10^{-3})}{2\times10^{-5}} = \dfrac{6.4\times10^{2}}{2\times10^{-5}} = 3.2\times10^{7}$

III. Substitution

1. $xyz - 4z = (3)(-4)(2) - 4(2) = -24 - 8 = -32$
2. $2x - y = 2(3) - (-4) = 6 + 4 = 10$
3. $x(y - 3z) = 3[-4 - 3(2)] = 3(-4 - 6) = 3(-10) = -30$
4. $\dfrac{5x - z}{xy} = \dfrac{5(3) - 2}{(3)(-4)} = \dfrac{13}{-12} = -\dfrac{13}{12}$
5. $3y^{2} - 2x + 4z = 3(-4)^{2} - 2(3) + 4(2) = 3(16) - 6 + 8 = 50$

IV. Linear equations in one variable

1. $6x - 48 = 6 \Rightarrow 6x - 48 + 48 = 6 + 48 \Rightarrow 6x = 54 \Rightarrow \dfrac{6x}{6} = \dfrac{54}{6} \Rightarrow x = 9$
2. $\dfrac{2}{3}x - 5 = x - 3 \Rightarrow 3\left(\dfrac{2}{3}x - 5\right) = 3(x - 3) \Rightarrow 2x - 15 = 3x - 9 \Rightarrow 2x - 15 + 15 = 3x - 9 + 15$
   $\Rightarrow 2x = 3x + 6 \Rightarrow 2x - 3x = 3x + 6 - 3x \Rightarrow -x = 6 \Rightarrow -1(-x) = -1(6) \Rightarrow x = -6$
3. $x = 12$
4. $8 - 4(x - 1) = 2 + 3(4 - x) \Rightarrow 8 - 4x + 4 = 2 + 12 - 3x \Rightarrow 12 - 4x = 14 - 3x \Rightarrow 12 - 4x - 12 = 14 - 3x - 12$
   $\Rightarrow -4x = 2 - 3x \Rightarrow -4x + 3x = 2 - 3x + 3x \Rightarrow -x = 2 \Rightarrow x = -2$

V. Formulas

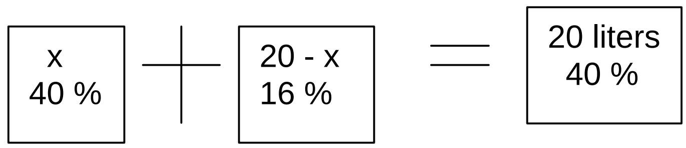
1. $PV = nRT \Rightarrow \dfrac{PV}{nR} = \dfrac{nRT}{nRT} \Rightarrow \dfrac{PV}{nR} = T$
2. $= 3x + \quad \Rightarrow y - 2 = \quad x = 2 - \quad \Rightarrow y - 2 = \quad x \Rightarrow \dfrac{y-}{3} = \dfrac{3}{3} \Rightarrow \dfrac{y-}{3} = x$
3. $r = \dfrac{C}{2\pi}$
4. $y = -\dfrac{5}{2}x + 5$
5. $y = hx + 4x \Rightarrow y = x(h + 4) \Rightarrow \dfrac{y}{h + 4} = \dfrac{x(h + 4)}{h + 4} \Rightarrow \dfrac{y}{h + 4} = x$

VI. Word Problems

1. Let x = “another number” forcing 2x + 5 = “One number.” x + 2x + 5 = 35 and x = 10. “One number” = 25 and “another number” = 10.

2. Let x = the dollars in the account paying 6% interest
   Then, 18,000 – x = the dollars in the account paying 8%.
   The interest dollars are calculated by multiplying the total dollars in the account by the interest rate.
   Hence: .06 x = the interest earned by the first account
   .08 (18,000 – x) = the interest earned by the second account. Adding up all the interest, .06x + .08(18,000 – x) = 1,290. Solving, x = 7,500. So, Ms. Jones has $7,500 in the account paying 6% interest and $10,500 in the account paying 8% interest.

3. Use the following buckets:



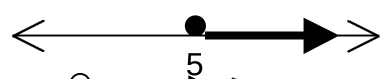
   From the diagram, we get the equation: .4x + .16 (20 – x) = 20(.4)
   x = 5 and the answer is 5 liters at 40% and 15 liters at 16%.

4. Let x = the number of burgers and 14 – x = the number of fries. To get the total amount of money spent, multiply the number of items by the cost of the item. 2.05 x = the total dollars spent on burgers and .85 (14 – x) = the total dollars spent on fries. The equation is: 2.05x + .85 (14 – x) = 19.10. Solving the equation, x = 6. Hence, she bought 6 burgers and 8 fries.

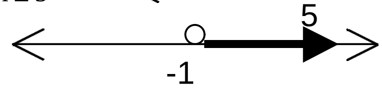
VII. Inequalities

Solve inequalities the same as equations with one exception. When both sides are multiplied or divided by a negative number, remember to switch the direction of the inequality.

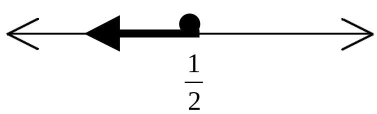
1. $2x - 7 \geq 3 \;\Rightarrow\; 2x - 7 + 7 \geq 3 + 7 \;\Rightarrow\; 2x \geq 10 \;\Rightarrow\; \frac{2x}{2} \geq \frac{10}{2} \;\Rightarrow\; x \geq 5$



2. $-5(2x+3) < 2x - 3 \Rightarrow -10x - 15 < 2x - 3 \Rightarrow -12x < 12 \Rightarrow x > -1$



3. $x \leq \frac{1}{2}$



VIII. Exponents & Polynomials

1. Add like terms: $(3x^2 - 5x - 6) + (5x^2 + 4x + 4) = 8x^2 - x - 2$

2. $\frac{(2a^{-5}b^4c^3)^{-2}}{(3a^3b^{-7}c^3)^2} = \frac{2^{-2}a^{10}b^{-8}c^{-6}}{3^2a^6b^{-14}c^6} = \frac{\frac{1}{4}a^{10-6}b^{-8-(-14)}c^{-6-6}}{9} = \frac{a^4b^6c^{-12}}{36} = \frac{a^4b^6}{36c^{12}}$

3. $(3x^0y^5z^6)(-2xy^3z^{-2}) = 3(-2)x^0x \cdot y^5y^3 \cdot z^6z^{-2} = -6xy^8z^4$

4. $(-a^5b^7c^9)^4 = (-1)^4a^{20}b^{28}c^{36} = a^{20}b^{28}c^{36}$

5. $(4x^2y^6z)^2(-x^{-2}y^3z^4)^6 = (16x^4y^{12}z^2)(x^{-12}y^{18}z^{24}) = 16x^{-8}y^{30}z^{26} = 16\frac{1}{x^8}y^{30}z^{26} = \frac{16y^{30}z^{26}}{x^8}$

6. $\frac{24x^4 - 32x^3 + 16x^2}{8x^2} = \frac{24x^4}{8x^2} - \frac{32x^3}{8x^2} + \frac{16x^2}{8x^2} = 3x^2 - 4x + 2$

7. $(x^2 - 5x)(2x^3 - 7) = 2x^5 - 7x^2 - 10x^3 + 35x = 2x^5 - 10x^3 - 7x^2 + 35x$

8. $\frac{26a^2b^{-5}c^9}{-4a^{-6}bc^9} = \frac{-13a^{2-(-6)}b^{-5-1}c^{9-9}}{2} = \frac{-13a^8b^{-6}c^0}{2} = -\frac{13a^8}{2b^6}$

9. $(5a+6)^2 = (5a+6)(5a+6) = 25a^2 + 30a + 30a + 36 = 25a^2 + 60a + 36$

IX. Factoring

Steps to factoring:
1. Always factor out the Greatest Common Factor (If possible).
2. Factor the first and third term.
3. Figure out the middle term.

1. $(x+6)(x-1)$
2. $(x+1)(x-6)$
3. $(2x-6)(2x+6)$, Difference of two squares
4. Sum of two squares requires the complex number system to factor. Not factorable.

5. $64x^4-4y^4=4(16x^4-y^4)=4(4x^2-y^2)(4x^2+y^2)=4(2x-y)(2x+y)(4x^2+y^2)$

6. Difference of two cubes: $a^3-b^3=(a-b)(a^2+ab+b^2)$. Let a = 2x and b = 3 and use the formula to get : $(2x-3)(4x^2+6x+9)$

7. $(7y+6)^2$
8. $(6x+3)(2x+1)$

X. Quadratic Equations

Steps:
1. Get zero on one side of the equals
2. Factor
3. Set each factor to zero
4. Solve for your variable

If you can not factor the equation and the quadratic is in the form $ax^2+bx+c=0$, then use the quadratic formula.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

1. $4a^2+9a+2=0 \quad\Rightarrow\quad (4a+1)(a+2)=0 \quad\Rightarrow\quad 4a+1=0 \text{ or } a+2=0 \quad\Rightarrow\quad a=-\frac{1}{4} \text{ or } a=-2$
2. 3, -3
3. $25x^2-6=30 \;\Rightarrow\; 25x^2-6-30=30-30 \;\Rightarrow\; 25x^2-36=0 \;\Rightarrow\; (5x-6)(5x+6)=0 \;\Rightarrow\; x=\frac{6}{5} \text{ or } x=-\frac{6}{5}$

4. 2, $-\frac{1}{3}$

5. The solution is given below:

$(3x+2)^2=16 \;\Rightarrow\; 9x^2+12x+4=16 \;\Rightarrow\; 9x^2+12x+4-16=16-16 \;\Rightarrow\; 9x^2+12x-12=0$

$\Rightarrow\; 3(3x+4x-4)=0 \;\Rightarrow\; 3(3x-2)(x+2)=0 \;\Rightarrow\; x=\frac{2}{3} \text{ or } x=-2$

6. $1\pm\sqrt{5}$

XI. Rational Expressions

1. Need to find a common denominator (factor denominators to see what you need), add, and then reduce (if possible) at the very end.

$$\frac{4}{2a-2}+\frac{3a}{a^2-a}=\frac{4}{2(a-1)}+\frac{3a}{a(a-1)}=\frac{4}{2(a-1)}\cdot\frac{a}{a}+\frac{3a}{a(a-1)}\cdot\frac{2}{2}=\frac{4a}{2a(a-1)}+\frac{6a}{2a(a-1)}=\frac{10a}{2a(a-1)}$$
$$=\frac{5}{a-1}$$

2. This problem uses the same technique as above. Be careful of the subtraction.

$$\frac{3}{x^2-1}-\frac{4}{x^2+3x+2}=\frac{3}{(x-1)(x+1)}-\frac{4}{(x+2)(x+1)}=\frac{3}{(x-1)(x+1)}\cdot\frac{(x+2)}{(x+2)}-\frac{4}{(x+2)(x+1)}\cdot\frac{(x-1)}{(x-1)}$$
$$=\frac{3x+6}{(x-1)(x+1)(x+2)}-\frac{4x-4}{(x-1)(x+1)(x+2)}=\frac{3x+6-4x+4}{(x-1)(x+1)(x+2)}=\frac{-x+10}{(x-1)(x+1)(x+2)}$$

3. To multiply fractions, factor and cancel first before multiplying.

$$\frac{6x-18}{3x^2+2x-8}\cdot\frac{12x-16}{4x-12}=\frac{6(x-3)}{(3x-4)(x+2)}\cdot\frac{4(3x-4)}{4(x-3)}=\frac{6\cancel{(x-3)}}{\cancel{(3x-4)}(x+2)}\cdot\frac{\cancel{4}\cancel{(3x-4)}}{\cancel{4}\cancel{(x-3)}}=\frac{6}{x+2}$$

4. Division is the same process with one extra step (invert & multiply): $\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}$. One other hint: $(1-x)=-(x-1)$ (Continues on next page)

$$\frac{16-x^2}{x^2+2x-8}\div\frac{x^2-2x-8}{4-x^2}=\frac{(4-x)(4+x)}{(x-2)(x+4)}\div\frac{(x-4)(x+2)}{(2-x)(2+x)}=\frac{(4-x)(4+x)}{(x-2)(x+4)}\cdot\frac{(2-x)(2+x)}{(x-4)(x+2)}$$
$$=\frac{-(x-4)(4+x)}{(x-2)(x+4)}\cdot\frac{-(x-2)(2+x)}{(x-4)(x+2)}=\frac{-\cancel{(x-4)}\cancel{(4+x)}}{\cancel{(x-2)}\cancel{(x+4)}}\cdot\frac{-\cancel{(x-2)}\cancel{(2+x)}}{\cancel{(x-4)}\cancel{(x+2)}}=1$$

5. Factor and Reduce to get x^2+x+1.

6. Find the Lowest common denominator (LCD) for all fractions (xy), then multiply the numerator and denominator by the LCD.

$$\frac{\frac{2}{x}-\frac{1}{y}}{\frac{1}{xy}}=\frac{xy}{xy}\cdot\frac{\left(\frac{2}{x}-\frac{1}{y}\right)}{\frac{1}{xy}}=\frac{2y-x}{1}=2y-x$$

7. Annihilate the denominators by multiplying both sides of the equation by the LCD $[(x-1)(x+1)4]$, solve the resulting, fractionless equation, and check answers in the original equation to insure that the denominators are not zero.

$$\frac{2}{x-1}+\frac{1}{x+1}=\frac{5}{4}\Rightarrow (x-1)(x+1)(4)\left[\frac{2}{x-1}+\frac{1}{x+1}\right]=\frac{5}{4}(x-1)(x+1)(4)\Rightarrow 2(x+1)(4)+(x-1)4=5(x-1)(x+1)$$
$$\Rightarrow 8x+8+4x-4=5x^2-5\Rightarrow 5x^2-12x-9=0\Rightarrow (5x+3)(x-3)=0\Rightarrow x=-\frac{3}{5}\text{ or } x=3$$

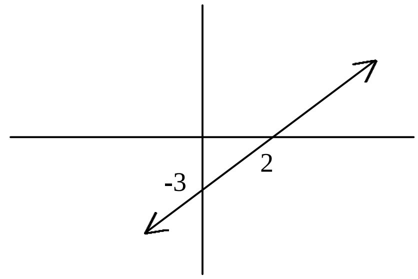
Since these answers do not make the denominator zero in the original equation, they are the solution.

8. k = -3

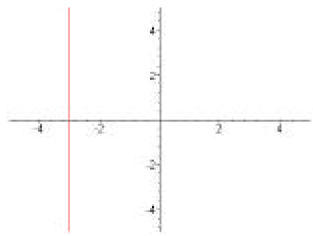
9. x = -8

XII. Graphing

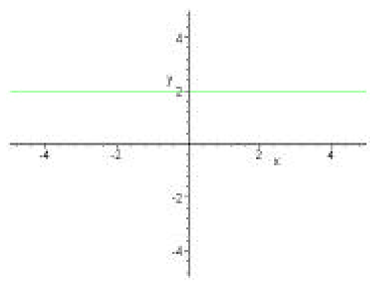
1. $3x - 2y = 6$



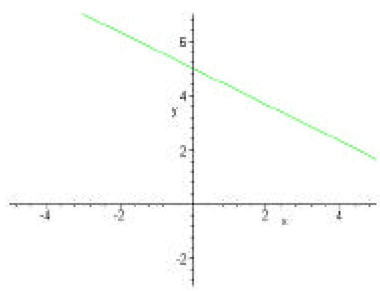
2. $x = -3$



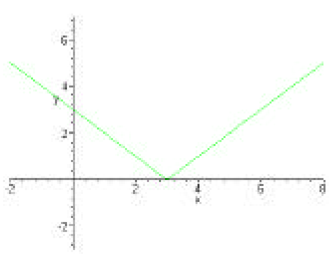
3. $y = 2$



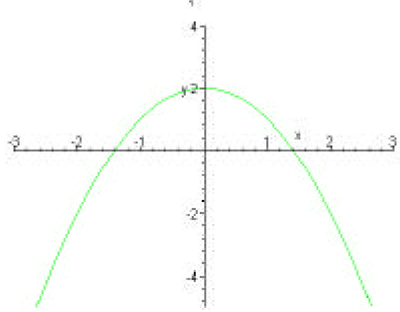
4. $y = \dfrac{-2}{3}x + 5$



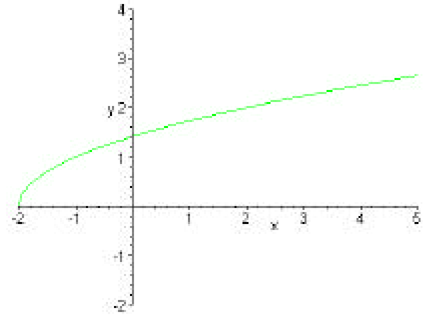
5. $y = |x - 3|$



6. $y = -x^2 + 2$



7. $y = \sqrt{x+2}$



XIII. Systems of Equations

The following are 2 dimensional linear equations. Each equation represents a line that can be graphed on the coordinate plane. The ultimate solution to a system of equations is for the lines to intersect in on point such as question #1 and #4.

Question #3 has two equations and one is a multiple of the other. Hence, both formulas graph the same line making the solution infinite.

The last possibility is in question #2. If you graph the lines in question #2, you will see that they are parallel and do not cross. This system has no solution.

1. The answer is x = 3 and y = 6. The work is below.

$$\begin{array}{lll} 2x - 3y = -12 & & 2x - 3y = -12 \\ x - 2y = -9 & \text{Multiply by -2} \rightarrow & \underline{-2x + 4y = \ \ 18} \\ & & y = \ \ 6 \quad \text{Now, substituting into the first equation} \\ & & \qquad 2x - 3(6) = -12 \qquad \Rightarrow \qquad x = 3 \end{array}$$

4. x = 1, y = 2

XIV. Radicals

Think of the index ($\sqrt[\text{index}]{\ }$) as a door person. If it is two, then two identical factors inside become one outside. Also, remember these properties:

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

1. $\left(\sqrt{8}\right)\left(\sqrt{10}\right) = \sqrt{8 \cdot 10} = \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 5} = 2 \cdot 2\sqrt{5} = 4\sqrt{5}$

2. $\sqrt[4]{\frac{81}{x^4}} = \frac{\sqrt[4]{81}}{\sqrt[4]{x^4}} = \frac{\sqrt[4]{3 \cdot 3 \cdot 3 \cdot 3}}{\sqrt[4]{x \cdot x \cdot x \cdot x}} = \frac{3}{x}$

3. $\sqrt{\frac{4}{3}} = \frac{\sqrt{4}}{\sqrt{3}} = \frac{\sqrt{2 \cdot 2}}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{\sqrt{3 \cdot 3}} = \frac{2\sqrt{3}}{3}$

4. $\sqrt{\frac{12}{18}} \cdot \sqrt{\frac{15}{40}} = \sqrt{\frac{12}{18} \cdot \frac{15}{40}} = \sqrt{\frac{5}{20}} = \frac{\sqrt{5}}{\sqrt{20}} = \frac{\sqrt{5}}{\sqrt{2 \cdot 2 \cdot 5}} = \frac{\sqrt{5}}{2\sqrt{5}} = \frac{1}{2}$

5. $\sqrt[3]{24x^3y^6} = \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y} = 2 \cdot x \cdot y \cdot y\sqrt[3]{3} = 2xy^2\sqrt[3]{3}$

6. Worked out below.

$$2\sqrt{18} - 5\sqrt{32} + 7\sqrt{162} = 2\sqrt{3 \cdot 3 \cdot 2} - 5\sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} + 7\sqrt{2 \cdot 9 \cdot 9} = 2 \cdot 3\sqrt{2} - 5 \cdot 2 \cdot 2\sqrt{2} + 7 \cdot 9\sqrt{2}$$
$$= 6\sqrt{2} - 20\sqrt{2} + 63\sqrt{2} = 49\sqrt{2}$$

7. $\frac{\sqrt{3}}{5-\sqrt{3}} = \left(\frac{\sqrt{3}}{5-\sqrt{3}}\right) \cdot \frac{\left(5+\sqrt{3}\right)}{\left(5+\sqrt{3}\right)} = \frac{5\sqrt{3}+3}{25-3} = \frac{5\sqrt{3}+3}{22}$

8. $\left(2\sqrt{3}+5\sqrt{2}\right)\left(3\sqrt{3}-4\sqrt{2}\right) = 6\sqrt{9} - 8\sqrt{6} + 15\sqrt{6} - 20\sqrt{4} = 18 - 8\sqrt{6} + 15\sqrt{6} - 40 = -22 + 7\sqrt{6}$