The Quadratic Formula

The solution to the quadratic equation \( ax^2 + bx + c = 0 \) \((a \neq 0)\) is

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},
\]

which we call the Quadratic Formula.

**PROOF**

1) \( ax^2 + bx + c = 0 \)  
   Start with the quadratic equation, then
   Subtract \( c \) from both sides of the equation

2) \( ax^2 + bx = -c \)  
   Divide both sides of the equation by \( a \)

3) \( \frac{ax^2}{a} + \frac{bx}{a} = -\frac{c}{a} \)  
   Simplify

4) \( x^2 + \frac{b}{a}x = -\frac{c}{a} \)  
   Now Complete the Square in \( x \)

5) \( \frac{1}{2} \cdot \frac{b}{a} = \frac{b}{2a} \); then \( \left( \frac{b}{2a} \right)^2 = \frac{b^2}{4a^2} \)  
   Take \( \frac{1}{2} \) of the coefficient of \( x \), then square this result, then add to both sides of the equation.

6) \( x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a} \)  
   Get a common denominator on the right hand side of the equation

7) \( x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c \cdot 4a}{a \cdot 4a} \)  
   Combine the two fractions on the right

8) \( x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{4ac}{4a^2} \)  
   Now FACTOR the left hand side of the equation, which is a perfect square trinomial

9) \( \left( x + \frac{b}{2a} \right)^2 = \frac{b^2 - 4ac}{4a^2} \)  
   Take the Square Root of both sides of the equation

10) \( \sqrt{\left( x + \frac{b}{2a} \right)^2} = \sqrt{\frac{b^2 - 4ac}{4a^2}} \)
11) \[ \sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{b^2 - 4ac}{4a^2}} \] Note: \[ \sqrt{Z^2} = |Z| \]

12) \[ \left|x + \frac{b}{2a}\right| = \sqrt{\frac{b^2 - 4ac}{4a^2}} \] Break the Absolute Value into 2 Cases: ±

13) \[ x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}} \] Break the radical into 2 radicals

14) \[ x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{\sqrt{4a^2}} \] Simplify the radical in the denominator

15) \[ x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2|a|} \] Assume that \( a > 0 \) to remove the absolute value. If \( a < 0 \), then divide both sides of the equation by \(-1\) at the start of the problem to make \( a > 0 \)

16) \[ x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a} \] Subtract \( \frac{b}{2a} \) from both sides of the equation

17) \[ x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} \] Combine the fractions

18) \[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

End Of Proof