Repeated Zeros of a Polynomial

Example 1

\[ f(x) = (x - 3)^1 = 0 \]
\[ x - 3 = 0 \]
\[ x = 3 \]

\[ x = 3 \] is a zero of multiplicity 1

We say \( x = 3 \) is a non-repeated zero

Note: A (Real) zero of multiplicity 1: Crosses the x-axis rather steeply like a line.

Definition: If the factor \((x - r)\) occurs more than once in the factorization of \( f \), then \( r \) is called a repeated zero, or multiple zero of \( f \).

Example 2

\[ f(x) = x^2 - 6x + 9 = 0 \]
\[ (x - 3)^2 = 0 \]
\[ (x - 3)(x - 3) = 0 \]
\[ x = 3 \text{ or } x = 3 \]

\[ x = 3 \] is a zero of multiplicity 2

Note: A (Real) zero of multiplicity 2: Just touches the x-axis but does not cross the x-axis

Gets "flat" near the x-axis

"Looks" like \( y = \pm x^2 \) at the x-intercept

Example 3

\[ f(x) = x^3 - 9x^2 + 27x - 27 = 0 \]
\[ (x - 3)^3 = 0 \]
\[ (x - 3)(x - 3)(x - 3) = 0 \]
\[ x = 3 \text{ or } x = 3 \text{ or } x = 3 \]

\[ x = 3 \] is a zero of multiplicity 3

Note: A (Real) zero of multiplicity 3: Crosses the x-axis

Gets "flat" near the x-axis

"Looks" like \( y = \pm x^3 \) at the x-intercept