Solving Equations & Inequalities Manual

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MAT1033 Intermediate Algebra
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Solving Equations & Inequalities Manual Outline
MAT1033 Intermediate Algebra

SOLVING EQUATIONS

I Linear Equations

____ 1) $5x - 2(x - 3) = 18$

____ 2) $\frac{1}{6}x + \frac{1}{3} = \frac{1}{2}(x - 2)$

____ 3) $0.02x + 0.08(10 - x) = 0.04(10)$

II Equations with Absolute Value

____ 4) $|2x + 3| = 13$

III Quadratic Equations

Factoring Method

____ 5) $x^2 + 2x - 15 = 0$

____ 6) $12x^2 + x - 6 = 0$

____ 7) **Square Root Method:** $(x - 3)^2 = 8$

____ 8) **Quadratic Formula:** $2x^2 + 3 = 6x$

____ 9) **Complete the Square Method:** $x^2 - 4x + 2 = 0$

IV Rational Equations

____ 10) $x - \frac{6}{x} = 1$

____ 11) $\frac{x}{x+1} + \frac{5}{x} = \frac{1}{x^2 + x}$

V Equations with Radicals

Square Root

____ 12) $\sqrt{x - 2} - 7 = -4$

____ 13) $\sqrt{x + 9} - \sqrt{x - 6} = 3$

____ 14) **Cube Root:** $\sqrt[3]{x + 5} = -2$

SOLVING INEQUALITIES

I Compound Inequalities

____ 15) $5 - 3x > 11$ or $4 \leq x - 1$

II Inequalities with Absolute Value

____ 16) $|2x - 1| \leq 5$

____ 17) $|x - 3| > 1$

Solving Equations

I. Linear Equations

How to Identify: An equation that can be written in the form $ax + b = 0$, where a, b, and c are real numbers and $a \neq 0$.

Solving A Linear Equation:

1. Clear the equation of fractions by multiplying both sides of the equation by the least common denominator (LCD) of all denominators in the equation.
2. Eliminate decimal points by multiplying both sides of the equation by 1, 100, or 1000 to move the decimal point.
3. Use the distributive property to remove grouping symbols such as parenthesis.
4. Combine like terms on each side of the equation.
5. Use the addition property of equality to rewrite the equation as an equivalent equation with variable terms on one side and numbers on the other side.
6. Use the multiplication property of equality to isolate the variable.
7. Check the proposed solution in the original equation.

1. $5x - 2(x-3) = 18$

$5x - 2x + 6 = 18$

$3x + 6 = 18$

$-6 \quad -6$

$\frac{3x}{3} = \frac{12}{3}$

$\boxed{x = 4}$ c

Check: $5(4) - 2(4-3) = 18$

$20 - 8 + 6 = 18$

$18 = 18$

$$2.\quad \frac{1}{6}x + \frac{1}{3} = \frac{1}{2}(x-2) \qquad \text{Check: } \frac{1}{6}(4) + \frac{1}{3} = \frac{1}{2}(4-2)$$

$$\overset{1}{\cancel{6}}\frac{1}{\cancel{6}}x + \overset{2}{\cancel{6}}\frac{1}{\cancel{3}} = \overset{3}{\cancel{6}}\frac{1}{\cancel{2}}(x-2) \qquad \frac{2}{3} + \frac{1}{3} = \frac{1}{2}(2)$$

$$x + 2 = 3(x-2) \qquad 1 = 1$$

$$x + 2 = 3x - 6$$

$$\underline{-2 \qquad\quad -2}$$

$$x = 3x - 8$$

$$\underline{-3x \qquad -3x}$$

$$\frac{-\cancel{2}x}{-\cancel{2}} = \frac{-8}{-\cancel{2}}$$

$$\boxed{x = 4}$$

$$3.\quad 0.02x + 0.08(10-x) = 0.04(10)$$

$$2x + 8(10-x) = 4(10)$$

$$2x + 80 - 8x = 40$$

$$80 - 6x = 40$$

$$\underline{-80 \qquad\quad -80}$$

$$\frac{-\cancel{6}x}{-\cancel{6}} = \frac{-40}{-\cancel{6}}$$

$$\boxed{x = \frac{20}{3}}$$

$$\text{Check: } 2\left(\frac{20}{3}\right) + 8\left(10 - \frac{20}{3}\right) = 40$$

$$13.33 + 80 - 53.33 = 40$$

$$40 = 40$$

II. Equations with Absolute Values

How to Identify:

If a is a positive number, then $|x| = a$ *is equivalent to* $x = a$ *or* $x = a$.

If a is negative, then $|x| = a$ *has no solution.*

If an absolute value equation is of the form $|x| = |y|$, *solve* $x = y$ *or* $x = -y$.

Solving Equations with Absolute Values:

1. Clear the equation of fractions.
2. Remove grouping symbols such as parentheses.
3. Simplify by combining like terms.
4. Write variable terms on one side and numbers on the other side using the addition property of inequality.
5. Isolate the variable using the multiplication property of inequality.

4. $|2x + 3| = 13$

$2x + 3 = -13$ OR $2x + 3 = 13$

$-3 \quad -3$ $\qquad$ $-3 \quad -3$

$\frac{2x}{2} = \frac{-16}{2}$ $\qquad$ $\frac{2x}{2} = \frac{10}{2}$

$\boxed{x = -8}$ OR $\boxed{x = 5}$

Check:

$|2(-8) + 3| = 13$ $\qquad$ $|2(5) + 3| = 13$

$|-16 + 3| = 13$ $\qquad$ $|10 + 3| = 13$

$|-13| = 13$ $\qquad$ $|13| = 13$

III. Quadratic Equations – Factoring Method

How to Identify:

A polynomial equation of degree 2. Use Zero-Factor Property to solve.

If a and b are real numbers and a • b = 0, then a = 0 or b = 0

Solving Quadratic Equations with the Factoring Method:

1. Write the equation in standard form so that one side of the equation is 0.
2. Factor the polynomial completely.
3. Set each factor containing a variable equal to 0.
4. Solve the resulting equations.
5. Check each solution in the original equation

5. $x^2 + 2x - 15 = 0$

$(x+5)(x-3) = 0$

$x + 5 = 0$ or $x - 3 = 0$

$-5 \quad -5 \qquad +3 \quad +3$

$x = -5$ or $x = 3$

$\{-5, 3\}$ ✓

Check: $(-5)^2 + 2(-5) - 15 = 0$

$25 - 10 - 15 = 0$

$0 = 0$

$(3)^2 + 2(3) - 15 = 0$

$9 + 6 - 15 = 0$

$0 = 0$

6. $12x^2 + x - 6 = 0$

$(3x - 2)(4x + 3) = 0$

$3x - 2 = 0$ or $4x + 3 = 0$

$+2 \quad +2 \qquad -3 \quad -3$

$\frac{3x}{3} = \frac{2}{3} \qquad \frac{4x}{4} = \frac{-3}{4}$

$x = \frac{2}{3}$ or $x = -\frac{3}{4}$

$\{\frac{2}{3}, -\frac{3}{4}\}$ ✓

Check: $12\left(\frac{2}{3}\right)^2 + \frac{2}{3} - 6 = 0$

$5\frac{1}{3} + \frac{2}{3} - 6 = 0$

$\frac{16}{3} + \frac{2}{3} - 6 = 0$

$\frac{18}{3} - 6 = 0$

$6 - 6 = 0$

$12\left(-\frac{3}{4}\right)^2 - \frac{3}{4} - 6 = 0$

$6\frac{3}{4} - \frac{3}{4} - 6 = 0$

$0 = 0$

III. Quadratic Equations – Square Root Method

How to Identify:
Is a polynomial equation of degree 2.
If a and b are real numbers and a • b = 0, then a = 0 or b = 0
Square Root Property – If b is a real number and if a^2=b, then a = ±√b

Solving Quadratic Equations with the Square Root Method:

1. Isolate the X^2 term
2. Square root both sides, use ±
3. Solve & Check

7. $(x-3)^2 = 8$

$\sqrt{(x-3)^2} = \pm\sqrt{8}$

$x - 3 = \pm 2\sqrt{2}$
$\quad +3 \quad +3$

$\boxed{x = 3 \pm 2\sqrt{2}}$ C ✓

Check: $(3 + 2\sqrt{2} - 3)^2 = 8$

$(2\sqrt{2})^2 = 8$

$4(2) = 8$

$(3 - 2\sqrt{2} - 3) = 8$

$(0 - 2\sqrt{2})^2 = |8|$

$-4(2) = |8|$

$-8 = |8|$

III. Quadratic Equations – Quadratic Formula

How to Identify:

Is a polynomial equation of degree 2. Use Zero-Factor Property to solve.

If a and b are real numbers and a • b = 0, then a = 0 or b = 0

If it's a quadratic equation written in the form $ax^2 + bx + c = 0$ that can't be factored.

Has the solutions: $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

Solving Quadratic Equations with the Quadratic Formula:

1. If the equation is in the form $(ax+b)^2 = c$, use the square root property and solve. If not, go to Step 2.
2. Write the equation in standard form: $ax^2 + bx + c = 0$.
3. Try to solve the equation by the factoring method. If not possible, go to Step 4.
4. Solve the equation by the quadratic formula.

8. $2x^2 + 3 = 6x$

$2x^2 - 6x + 3 = 0$

$x = \dfrac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(3)}}{2(2)}$

$x = \dfrac{6 \pm \sqrt{12}}{4}$

$x = \dfrac{6 \pm 2\sqrt{3}}{4}$

$\boxed{x = \dfrac{3 \pm \sqrt{3}}{2}}$ Q ✓

III. Quadratic Equations – Complete the Square Method

How to Identify:

Is a polynomial equation of degree 2. Use Zero-Factor Property to solve.
If a and b are real numbers and a • b = 0, then a = 0 or b = 0
If it's a quadratic equation written in the form $ax^2 + bx + c = 0$ that can't be factored.

Solving Quadratic Equations with the Complete the Square Method:

1. If the coefficient of x^2 is 1, go to step 2. Otherwise, divide both sides of the equation by the coefficient of x^2.
2. Isolate all variable terms on one side of the equation.
3. Complete the square for the resulting binomial by adding the square of half of the coefficient of x to both sides of the equation.
4. Factor the resulting perfect square trinomial and write it as the square of a binomial.
5. Use the square root property to solve for x.

9. $x^2 - 4x + 2 = 0$

$x^2 - 4x + 4 = -2 + 4$

$(x - 2)^2 = 2$

$\sqrt{(x-2)^2} = \pm\sqrt{2}$

$x - 2 = \pm\sqrt{2}$
$+2 \quad +2$

$\boxed{x = 2 \pm \sqrt{2}}$ C

IV. Rational Equations

How to Identify: Equation containing rational expressions.

Solving Rational Equations:

1. Multiply both sides of the equation by the LCD of all rational expressions in the equation to eliminate denominators.
2. Simplify both sides.
3. Solve resulting equation.
4. Set to = 0
4. Check the answer so denominator ≠ 0

10. $x - \frac{6}{x} = 1$

$\frac{\cancel{x}}{1} \cdot \frac{x}{1} - \frac{6}{\cancel{x}} \cdot \frac{x}{1} = \frac{1}{1} \cdot \frac{x}{1}$

$x^2 - 6 = x$

$x^2 - x - 6 = 0$

$(x+2)(x-3) = 0$

$x + 2 = 0$ OR $x - 3 = 0$

$-2 \quad -2 \qquad +3 \quad +3$

$\boxed{x = -2 \text{ OR } x = 3}$

Check: $\frac{-2}{1} - \frac{6}{-2} = 1$

$\frac{-2}{1} \cdot \frac{-2}{1} - \frac{6}{\cancel{-2}} \cdot \frac{\cancel{-2}}{1} = \frac{1}{1} \cdot \frac{-2}{1}$

$4 - 6 = -2$

$\frac{3}{1} - \frac{6}{-2} = \frac{1}{1}$

11. $\frac{x}{x+1} + \frac{5}{x} = \frac{1}{x^2+x}$

$\frac{x}{\cancel{x+1}} \cdot \frac{x\cancel{(x+1)}}{1} + \frac{5}{\cancel{x}} \cdot \frac{\cancel{x}(x+1)}{1} = \frac{x(x+1)}{x(x+1)}$

$x^2 + 5(x+1) = 1$

$x^2 + 5x + 5 = 1$

$-1 \qquad -1$

$x^2 + 5x + 4 = 0$

$(x+1)(x+4) = 0$

$x + 1 = 0$ OR $x + 4 = 0$

$-1 \quad -1 \qquad -4 \quad -4$

$x = -1$ OR $\boxed{x = -4}$

$\varnothing$

$x \neq -1$

restrictions: $x \neq -1$

Check: $\frac{-4}{-4+1} + \frac{5}{-4} = \frac{1}{-4^2+4}$

$\frac{-4}{-3} + \frac{-5}{4} = \frac{1}{16-4}$

$\frac{4}{3} + \frac{-5}{4} = \frac{1}{12}$

$\frac{4}{\cancel{3}} \cdot \frac{\overset{4}{\cancel{12}}}{1} + \frac{-5}{\cancel{4}} \cdot \frac{\overset{3}{\cancel{12}}}{1} = \frac{1}{\cancel{12}} \cdot \frac{\cancel{12}}{1}$

$16 - 15 = 1$

true

V. Equations with Radicals – Square Root

How to Identify:

Equations that contain radical expressions and use the power rule to help solve the equation: If both sides of and equation are raised to the same power, all solutions of the original are among the solutions of the new equation.

Solving Equations with Radicals – Square Root:

1. Isolate one radical on one side of the equation.
2. Raise each side of the equation to a power equal to the index of the radical and simplify.
3. If the equation still contains a radical term, repeat Steps 1 and 2. If not, solve the equation.
4. Check all proposed solutions in the original equation.

12. $\sqrt{x-2} - 7 = -4$ Check: $\sqrt{11-2} - 7 = -4$

$\quad +7 \quad +7$ $\sqrt{9} - 7 = -4$

$\sqrt{x-2} = 3$ $3 - 7 = -4$

$(\sqrt{x-2})^2 = (3)^2$ $-4 = -4$

$x - 2 = 9$

$\quad +2 \quad +2$

$\boxed{x = 11}$

13. $\sqrt{x+9} - \sqrt{x-6} = 3$ Check: $\sqrt{7+9} - \sqrt{7-6} = 3$

$(\sqrt{x+9})^2 = (\sqrt{x-6} + 3)^2$ $\sqrt{16} - \sqrt{1} = 3$

$x + 9 = (\sqrt{x-6} + 3)(\sqrt{x-6} + 3)$ $4 - 1 = 3$

$x + 9 = \sqrt{x-6}^2 + 3\sqrt{x-6} + 3\sqrt{x-6} + 9$ $3 = 3$

$x + 9 = x - 6 + 6\sqrt{x-6} + 9$

$x + 9 = x + 3 + 6\sqrt{x-6}$

$-x - 3 \quad -x - 3$

$6 = 6\sqrt{x-6}$

$(6)^2 = (6\sqrt{x-6})^2$

$\frac{36}{36} = \frac{36(x-6)}{36}$

$1 = x - 6$

$+6 \quad +6$

$7 = x$ $\boxed{x = 7}$

V. Equations with Radicals – Cube Root

How to Identify:

Equations that contain radical expressions and use the power rule to help solve the equation: If both sides of and equation are raised to the same power, all solutions of the original are among the solutions of the new equation.

Solving Equations with Radicals – Cube Root:

1. Isolate one radical on one side of the equation.
2. Raise each side of the equation to a power equal to the index of the radical and simplify.
3. If the equation still contains a radical term, repeat Steps 1 and 2. If not, solve the equation.
4. Check all proposed solutions in the original equation.

14. $\sqrt[3]{x+5} = -2$ Check: $\sqrt[3]{-13+5} = -2$

$(\sqrt[3]{x+5})^3 = (-2)^3$ $\sqrt[3]{-8} = -2$

$x+5 = -8$

$-5 \quad -5$

$\boxed{x = -13}$ C

Solving Inequalities

I. Compound Inequalities

How to Identify:

Two inequalities joined by the words ***and*** *or* ***or****.*

The solution set of a compound inequality formed by the word ***and*** *is the intersection of the solution sets of the two equations.*

The solutions set of a compound inequality formed by the word ***or*** *is the union of the solutions sets of the two inequalities.*

Solving Compound Inequalities

1. Solve each inequality by clearing equations of fractions.
2. Remove grouping symbols such as parentheses.
3. Simplify by combining like terms.
4. Write variable terms on one side and numbers on the other side using the addition property, distributive property and multiplication and then simplify.
5. Graph the solution set.

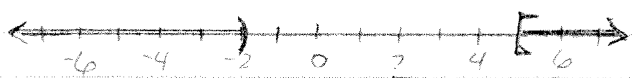
15. $5 - 3x > 11$ or $4 \le x - 1$

$\quad -5 \quad -5 \qquad +1 \quad +1$

$-3x > 6 \qquad 5 \le x$

$-3 \quad -3$

$x > -2$ or $x \ge 5$



$(-\infty, -2) \cup [5, \infty)$

II. Inequalities with Absolute Value

How to Identify:

If a is a positive number, then $|x| < a$ *is equivalent to* $-a < x < a$.

If a is a positive number, then $|x| < a$ *is equivalent to* $-a < x > a$.

Solving Inequalities with Absolute Value

1. Solve each inequality by clearing equations of fractions.
2. Remove grouping symbols such as parentheses.
3. Simplify by combining like terms.
4. Write variable terms on one side and numbers on the other side using the addition property, distributive property and multiplication and then simplify.
5. Apply property of Absolute Value Inequalities and solve resulting inequalities.
6. Graph Solution Set

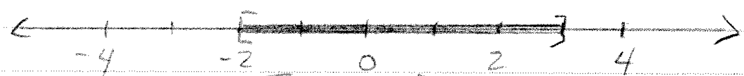
16. $|2x - 1| \leq 5$

$2x - 1 \geq -5$ and $2x - 1 \leq 5$

$+1 \quad +1$ $\quad\quad$ $+1 \quad +1$

$\frac{2x}{2} \geq \frac{-4}{2}$ $\quad\quad$ $\frac{2x}{2} \leq \frac{6}{2}$

$x \geq -2$ and $x \leq 3$



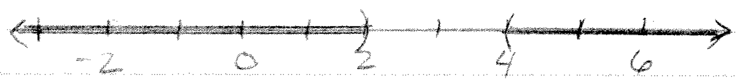
$[-2, 3]$ c

17. $|x - 3| > 1$

$x - 3 < -1$ OR $x - 3 > 1$

$+3 \quad +3$ $\quad\quad$ $+3 \quad +3$

$x < 2$ OR $x > 4$



$(-\infty, 2) \cup (4, \infty)$

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