

# Solving Equations

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# Inequalities Manual

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MAT 1033 Intermediate Algebra

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## Solving Equations & Inequalities Manual Outline – 20 Problems

### SOLVING EQUATIONS

#### **I Linear Equations**

- ✓ 1)  $5x - 2(x - 3) = 18$   
✓ 2)  $\frac{1}{6}x + \frac{1}{3} = \frac{1}{2}(x - 2)$   
✓ 3)  $0.02x + 0.08(10 - x) = 0.04(10)$   
✓ 4)  $5(x - 2) + x - 3 = 6x + 1$

#### **II Equations with Absolute Value**

- ✓ 5)  $|x - 4| = 7$   
✓ 6)  $|2x + 3| - 1 = 12$

#### **III Quadratic Equations**

##### **Factoring Method**

- ✓ 7)  $x^2 + 2x - 15 = 0$   
✓ 8)  $x^2 + 13x = 48$   
✓ 9)  $12x^2 + x - 6 = 0$   
✓ 10) **Square Root Method:**  $(x - 3)^2 = 8$   
✓ 11) **Quadratic Formula:**  $2x^2 + 3 = 6x$   
✓ 12) **Complete the Square Method:**  $x^2 - 4x + 2 = 0$

#### **IV Rational Equations**

- ✓ 13)  $x - \frac{6}{x} = 1$   
✓ 14)  $\frac{x}{x+1} + \frac{5}{x} = \frac{1}{x^2 + x}$

#### **V Equations with Radicals**

##### **Square Root**

- ✓ 15)  $\sqrt{x - 2} - 7 = -4$   
✓ 16)  $\sqrt{x + 9} - \sqrt{x - 6} = 3$   
✓ 17) **Cube Root:**  $\sqrt[3]{x + 5} = -2$

### SOLVING INEQUALITIES

- ✓ 18) **Compound Inequalities:**  $5 - 3x > 11$  or  $4 \leq x - 1$

##### **Inequalities with Absolute Value**

- ✓ 19)  $|2x - 1| \leq 5$   
19)  $|x - 3| > 1$

## I. Linear Equations

Identify: This equation can be put into the form:  $ax + b = 0$

Strategies

- 1) Get rid of fractions
- 2) Remove the parentheses
- 3) Combined like terms
- 4) Get the "x" by itself
- 5) Check your answers

1.  $5x - 2(x - 3) = 18$

$$5x - 2x + 6 = 18$$

$$3x + 6 = 18$$

$$3x = 12$$

$$\boxed{x = 4}$$

distribute the -2

Combine like terms

sub 6

divide by 3

Check:

$$5(4) - 2(4 - 3) = 18$$

$$20 - 2(1) = 18$$

$$20 - 2 = 18$$

$$18 = 18 \quad \checkmark$$

2.  $\frac{1}{6}x + \frac{1}{3} = \frac{1}{2}(x - 2)$

LCD = 6

$$(6)\frac{1}{6}x + (6)\frac{1}{3} = (6)\frac{1}{2}(x - 2)$$

$$x + 2 = 3(x - 2)$$

$$x + 2 = 3x - 6$$

$$8 = 2x$$

$$\boxed{x = 4}$$

mult. both sides by LCD

distribute 3

Add 6 + sub x

Divide by 2

Check:

$$\frac{1}{6}(4) + \frac{1}{3} = \frac{1}{2}(4-2)$$

$$\frac{2}{3} + \frac{1}{3} = 1$$

$$1 = 1 \quad \checkmark$$

3.  $0.02x + 0.08(10-x) = 0.04(10)$

$$\frac{1}{50}x + \frac{2}{25}(10-x) = \frac{1}{25}(10)$$

LCD=50

$$(50)\frac{1}{50}x + (50)\frac{2}{25}(10-x) = (50)\frac{1}{25}(10)$$

Mult. both sides  
by LCD

$$x + 4(10-x) = 2(10)$$

distribute 10

$$x + 40 - 4x = 20$$

combine like terms

$$-3x + 40 = 20$$

sub. 40

$$-3x = -20$$

Divide by -3

$$x = \frac{20}{3} \checkmark$$

Check:

$$.02\left(\frac{20}{3}\right) + .08\left(10 - \frac{20}{3}\right) = .04(10)$$

$$\frac{2}{15} + \frac{4}{15} = \frac{2}{5}$$

$$\frac{2}{5} = \frac{2}{5} \quad \checkmark$$

4.  $5(x-2) + x-3 = 6x+1$

distribute 5

$$5x-10+x-3 = 6x+1$$

combine like terms

$$6x-13 = 6x+1$$

Sub 6x + Add 13

$$0 \neq 14$$

No solution  $\checkmark$

Can Not Check

## II. Equations with Absolute value

Identify: This is an equation that has the absolute value bars in it

Strategies:

- 1) Get the absolute value bars by themselves.
- 2) Write the equation down twice
- 3) Set one equal to a positive number and the other equal to a negative number
- 4) Solve each equation
- 5) Check your answers

5.  $|x-4|=7$

$$x-4=7$$

$$x=11$$

$$x-4=-7$$

$$x=-3$$

set one = 7

and one = -7

Add 4

Check:

$$|11-4|=7$$

$$|7|=7$$

$$7=7 \checkmark$$

$$|-3-4|=7$$

$$|-7|=7$$

$$7=7 \checkmark$$

6.  $|2x+3|-1=12$

$$|2x+3|=13$$

$$2x+3=13$$

$$2x=10$$

$$x=5$$

$$2x+3=13$$

$$2x=-16$$

$$x=-8$$

Get "1" by itself

set one = 13

set one = -13

Sub. 3

divide by 2

Check:

$$|2(5)+3|-1=12$$

$$|13|-1=12$$

$$12=12 \checkmark$$

$$|2(-8)+3|-1=12$$

$$|-13|-1=12$$

$$12=12 \checkmark$$

### III. Quadratic Equations

Identify: This equation has an " $x^2$ " in it and its form is:  $ax^2 + bx + c = 0$

#### Factoring Method

Strategies.

- 1) set it equal to zero
- 2) Take out the GCM
- 3) Factor using ( )
- 4) set them both equal to zero
- 5) solve for each equation
- 6) check your answers

7.  $x^2 + 2x - 15 = 0$

$$(x+5)(x-3) = 0$$

$$x+5 = 0$$

$$x = -5$$

$$x-3 = 0$$

$$x = 3$$

Factor using ( )  
sub 5 + Add 3

Check:

$$(-5)^2 + 2(-5) - 15 = 0$$

$$25 - 10 - 15 = 0$$

$$15 - 15 = 0$$

$$0 = 0 \checkmark$$

$$(3)^2 + 2(3) - 15 = 0$$

$$9 + 6 - 15 = 0$$

$$15 - 15 = 0$$

$$0 = 0 \checkmark$$

8.  $x^2 + 13x = 48$

$$x^2 + 3x - 48 = 0$$

$$(x+16)(x-3) = 0$$

$$x+16 = 0$$

$$x = -16$$

$$x-3 = 0$$

$$x = 3$$

set it = 0

Factor using ( )  
sub 16 + Add 3

Check:

$$(-16)^2 + 13(-16) = 48$$

$$256 - 208 = 48$$

$$48 = 48 \checkmark$$

$$3^2 + 13(3) = 48$$

$$9 + 39 = 48$$

$$48 = 48 \checkmark$$

9.  $12x^2 + x - 6 = 0$

$$(4x+3)(3x-2) = 0$$

$$4x+3=0 \quad 3x-2=0$$

$$4x = -3 \quad 3x = 2$$

$$x = -3/4 \quad \checkmark \quad x = 2/3$$

Factor using (X)

set them both = 0

sub 3 + Add 2

divide by 4 + 3

Check:

$$12(-3/4)^2 + (-3/4) - 6 = 0$$

$$27/4 + -27/4 = 0$$

$$0 = 0 \checkmark$$

$$12(2/3)^2 + (2/3) - 6 = 0$$

$$16/3 + -16/3 = 0$$

$$0 = 0 \checkmark$$

## Square Root Method

Strategies:

- 1) get the  $( )^2$  by itself
- 2) take the square root of each side
- 3) solve for "X" and simplify  $\sqrt{\quad}$
- 4) try and check your answer

10.  $(x-3)^2 = 8$

$$\sqrt{(x-3)^2} = \sqrt{8}$$

$$x-3 = \pm\sqrt{8}$$

$$x = 3 \pm \sqrt{8}$$

$$x = 3 \pm \sqrt{4 \cdot 2}$$

$$x = 3 \pm 2\sqrt{2}$$

Take the  $\sqrt{\quad}$  of both sides  
rewrite  
Add 3

simplify the  $\sqrt{\quad}$

Check:

$$((3+2\sqrt{2})-3)^2 = 8$$

$$(2.8284)^2 = 8$$

$$8 = 8 \checkmark$$

$$((3-2\sqrt{2})-3)^2 = 8$$

$$(-2.8284)^2 = 8$$

$$8 = 8 \checkmark$$



## Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

### Strategies

- 1) set the equation equal to zero
- 2) substitute  $ax^2 + bx + c = 0$  in the equation  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

3) simplify

11.  $2x^2 + 3 = 6x$

$$2x^2 - 6x + 3 = 0$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(3)}}{2(2)}$$

$$x = \frac{6 \pm \sqrt{12}}{4}$$

$$x = \frac{6 \pm \sqrt{4 \cdot 3}}{4}$$

$$x = \frac{3 \pm 2\sqrt{3}}{2} \quad \text{①}$$

$$x = \frac{3 \pm \sqrt{3}}{2}$$

$$x = \frac{3 \pm \sqrt{3}}{2}$$

set the equation = 0  
substitute in a, b, c

Solve the discriminant

Simplify the  $\sqrt{\quad}$

simplify

## Complete the square Method

Strategies:

- 1) set the equation equal to zero
- 2) bring C over to the other side
- 3) take half of b then square it
- 4) add that number to both sides
- 5) Factor the equation
- 6) do the square root method

$$12. x^2 - 4x + 2 = 0$$

$$x^2 - 4x = -2$$

sub 2

$$x^2 - 4x + 4 = -2 + 4$$

half then sq. -4

$$x^2 - 4x + 4 = 2$$

then add it to both side

$$(x-2)(x-2) = 2$$

factor

$$\sqrt{(x-2)^2} = \sqrt{2}$$

$\sqrt{\quad}$  both sides

$$x-2 = \pm\sqrt{2}$$

$$x = 2 \pm \sqrt{2} \quad \checkmark$$

Add 2

Check:

$$(2 + \sqrt{2})^2 - 4(2 + \sqrt{2}) + 2 = 0$$

$$(11.656) - 13.656 + 2 = 0$$

$$0 = 0 \quad \checkmark$$

$$(2 - \sqrt{2})^2 - 4(2 - \sqrt{2}) + 2 = 0$$

$$(1.34314) - 2.34314 + 2 = 0$$

$$0 = 0 \quad \checkmark$$

## IV. Rational Equations

Identify: This equation has no Absolute value or square roots in the denominator.

Strategies:

- 1) Multiply both sides by the LCD
- 2) Set the equation equal to zero
- 3) Factor by using (X)
- 4) set both equations equal to zero
- 5) solve for "x"
- 6) Check your answers

$$13. \quad x - \frac{6}{x} = 1$$

$$\text{LCD} = x$$

$$(x) \cdot x - (x) \frac{6}{x} = (x) \cdot 1$$

Mult by LCD

$$x^2 - 6 = x$$

$$x^2 - x - 6 = 0$$

set = to 0

$$(x-3)(x+2) = 0$$

Factor

$$x-3=0$$

$$x+2=0$$

set both = 0

$$x=3$$

$$x=-2$$

Add 3 + sub 2

Check:

$$3 - \frac{6}{3} = 1$$

$$-2 - \frac{6}{-2} = 1$$

$$3 - 2 = 1$$

$$-2 + 3 = 1$$

$$1 = 1 \checkmark$$

$$1 = 1 \checkmark$$

$$14. \frac{x}{x+1} + \frac{5}{x} = \frac{1}{x^2+x}$$

$$\text{LCD} = x(x+1)$$

$$x(x+1) \left( \frac{x}{x+1} + \frac{5}{x} \right) = \frac{1}{x^2+x} x(x+1)$$

Mult. both sides by LCD

$$x^2 + 5x + 5 = 1$$

$$\text{set} = 0$$

$$x^2 + 5x + 4 = 0$$

$$(x+4)(x+1) = 0$$

Factor

$$x+4=0$$

$$x+1=0$$

set both eq = 0

$$x = -4$$

$$\cancel{x = -1}$$

sub. 4 + 1

Check

$$\frac{-4}{-4+1} + \frac{5}{-4} = \frac{1}{-4^2+(-4)}$$

$$\frac{4}{3} - \frac{5}{4} = \frac{1}{12}$$

$$\frac{1}{12} = \frac{1}{12} \checkmark$$

$$\frac{-1}{-1+1} + \frac{5}{-1} = \frac{1}{(-1)^2+(-1)}$$

$$\emptyset = \emptyset \checkmark$$

✓

## IV. Equations with Radicals

Identify: An equation with a radical in it.

Square Root

Strategies:

- 1) Get the Radical by itself
- 2) square both sides
- 3) solve for X
- 4) Check your answer

$$\begin{aligned} 15. \sqrt{x-2} - 7 &= -4 \\ (\sqrt{x-2})^2 &= (3)^2 \\ x-2 &= 9 \\ x &= 11 \text{ C} \end{aligned}$$

get  $\sqrt{\quad}$  by itself  
sq both sides,  
solve for "x"

Check:

$$\begin{aligned} \sqrt{11-2} - 7 &= -4 \\ \sqrt{9} - 7 &= -4 \\ 3 - 7 &= -4 \\ -4 &= -4 \checkmark \end{aligned}$$

$$\begin{aligned} 16. \sqrt{x+9} - \sqrt{x-6} &= 3 \\ (\sqrt{x+9})^2 &= (3 + \sqrt{x-6})^2 \\ x+9 &= 9 + 6\sqrt{x-6} + x-6 \\ x+9 &= 3 + x + 6\sqrt{x-6} \\ (9)^2 &= (\sqrt{x-6})^2 \\ 1 &= x-6 \\ x &= 7 \text{ C} \end{aligned}$$

get radical by itself  
square both sides.  
Combine like terms  
sub  $x + 3$  then divide by 6  
sq. both sides  
Add 6

Check

$$\begin{aligned} \sqrt{7+9} - \sqrt{7-6} &= 3 \\ 4 - 1 &= 3 \end{aligned}$$

$$3 = 3 \checkmark$$

## Cube Root

Strategies:

- 1) The same as square root
- 2) But with  $\sqrt[3]{\quad}$  (Cube root)

$$\begin{aligned} 17. \quad \sqrt[3]{x+5} &= -2 \\ (\sqrt[3]{x+5})^3 &= (-2)^3 \\ x+5 &= -8 \\ x &= -13 \quad \checkmark \end{aligned}$$

cube both sides

Sub 5

Check:

$$\begin{aligned} \sqrt[3]{-13+5} &= -2 \\ \sqrt[3]{-8} &= -2 \\ -2 &= -2 \quad \checkmark \end{aligned}$$

## Compound Inequalities

Identify: an equation with an inequality sign in it (such as  $>$   $<$   $\geq$   $\leq$ )

Strategies:

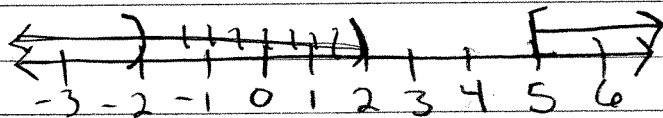
- 1) Get the "x" by itself
- 2) Solve
- 3) Graph
- 4) write interval notation
- 5) Check

$$18. 5 - 3x > 11 \quad \text{or} \quad 4 \leq x - 1$$

$$-3x > 6 \quad \text{or} \quad 5 \leq x$$

$$x < -2 \quad \cup \quad x \geq 5$$

sub 5 + Add 1  
divide by -3 + switch sig



graph

Interval Notation =  $(-\infty, 2) \cup [5, \infty)$

Check

$$5 - 3(2) > 11$$

$$\text{or} \quad 4 \leq 5 - 1$$

$$5 - 6 > 11$$

$$4 \leq 4 \quad \checkmark$$

$$-1 > 11 \quad \times$$

## Inequalities with Absolute Value

Identify: An equation that has Absolute value bars and inequality signs.

Strategies:

- 1) Separate the equation
- 2) Make one equal to a negative number (switch sign) and one equal to a positive.
- 3) Solve for "x"
- 4) Graph
- 5) Interval notation.

$$19. |2x - 1| \leq 5$$

$$2x - 1 \geq -5$$

$$2x \geq -4$$

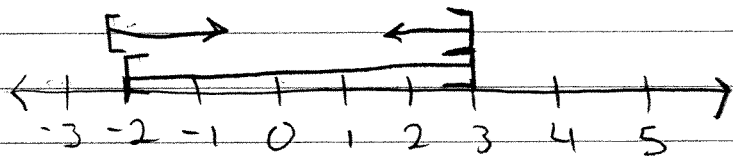
$$x \geq -2$$

$$2x - 1 \leq 5$$

$$2x \leq 6$$

$$x \leq 3$$

make one positive  
and one negative  
Add 1  
divide by 2



graph

$$[-2, 3]$$

interval Notation

Check:

$$|2(-2) - 1| \leq 5$$

$$|-4 - 1| \leq 5$$

$$|-5| \leq 5$$

$$5 \leq 5 \checkmark$$

$$|2(3) - 1| \leq 5$$

$$|6 - 1| \leq 5$$

$$|5| \leq 5$$

$$5 \leq 5 \checkmark$$



$$20. |x-3| > 1$$

$$x-3 < -1$$

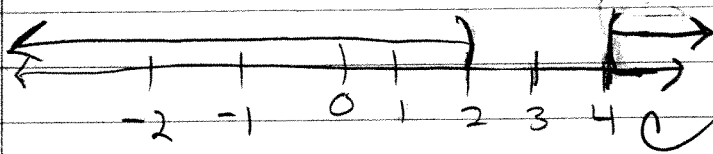
$$x < 2$$

U

$$x-3 > 1$$

$$x > 4$$

make one Pos. + one neg.  
Add 3



graph

$$(-\infty, 2) \cup (4, \infty)$$

interval notation

Check:

$$|2-3| > 1$$

$$|-1| > 1$$

$$1 > 1 \quad \checkmark$$

$$|4-3| > 1$$

$$|1| > 1$$

$$1 > 1 \quad \checkmark$$