

Jessica
Alex

Solving Equations Manual
MAC1105 – Fall 2006

Name: _____
Class Days / Time: MWF @ 11am

I **Linear Equations**

✓ 1) $\frac{2}{3}x - \frac{1}{4} = \frac{5}{6}(x - 2)$

II **Equations with Absolute Value**

✓ 2) $|2x - 3| + 1 = 12$

III **Quadratic Equations**

✓ 3) Factor Method: $x^2 - 3x = 18$

✓ 4) Square Root Method: $(x - 3)^2 = 12$

✓ 5) Quadratic Formula: $3x^2 - 2x - 4 = 0$

✓ 6) Complete the Square: $x^2 - 6x + 12 = 0$

IV **Cubic Equation** (Find One Real solution only)

✓ 7) $2(x - 4)^3 + 1 = 11$

V **Rational Equations**

✓ 8) $\frac{x}{x + 1} + \frac{2}{x} = 5$

VI **Equations with Radicals**

✓ 9) Square Root: $x = \sqrt{x + 7} + 5$

✓ 10) Cube Root: $\sqrt[3]{4x - 1} = 2$

VII **Higher Order Polynomials** (degree > 3)

✓ 11) $x^4 - x^3 - 13x^2 + 25x - 12 = 0$

✓ 12) $2x^4 - 5x^3 - 7x^2 + 10x + 6 = 0$

VIII **Exponential Equations**

✓ 13) $2^{x-4} = 25$

✓ 14) $200 = 100e^{0.5x}$

IX **Logarithmic Equations**

✓ 15) $\log(x - 5) = 2$

✓ 16) $\ln(x + 1) = 3$

Solving Equations Manual - Required Format

1. This Outline Sheet serves as your Cover Sheet and Table of Contents.
2. Start each equation on a new page, front side only. Number your equations.
3. **Outline a General Strategy** to solve the given type of equation.
4. **Solve each equation algebraically (by hand)**. Show your work solving the equation.
5. **Check your solution(s)** - either numerically or graphically.

1.

Solving Equations Manual

1

I. Linear Equations

★ Definition:

A Linear Equation is an equation of the form $ax+b=0$

Goal:

Solve for x ; find the values of (x)

Steps:

(1) First, we need to distribute:

$$\frac{5}{6}(x-2)$$

(2) Then combine like terms; such as x to x 's + numbers to numbers

(3) Make sure to get the least common factors for each side

(4) Solve for x and eliminate the coefficient of " x ".

Solve Problem:

$$\frac{2}{3}x - \frac{1}{4} = \frac{5}{6}(x-2)$$

$$\frac{2}{3}x - \frac{1}{4} = \frac{5}{6}x - \frac{10}{6}$$

$$\begin{array}{r} -\frac{1}{4} = \frac{5}{6}x - \frac{2}{3}x - \frac{10}{6} \\ +\frac{10}{6} \end{array}$$

$$\begin{array}{r} -\frac{1}{4} = \frac{3}{12} \\ +\frac{10}{6} = \frac{20}{12} \end{array}$$

$$-\frac{1}{4} + \frac{10}{6} = \frac{5}{6}x - \frac{2}{3}x \rightarrow$$

$$\frac{17}{12}$$

$$\frac{17}{12} = \frac{5}{6}x - \frac{2}{3}x$$

$$\frac{2}{3}x = \frac{4}{6}$$

$$\frac{6}{1} \cdot \frac{17}{12} = \frac{6}{6}x \cdot \frac{6}{1}$$

$$\frac{102}{12} = x$$

$$x = \frac{17}{2}$$

✓ Check Problem:

$$f(x) = \frac{17}{2}$$

$$\frac{2}{3}x - \frac{1}{4} = \frac{5}{6}(x-2)$$

$$\frac{68}{12} - \frac{3}{12} = \frac{85}{12} - \frac{20}{12}$$

$$\frac{2}{3}\left(\frac{17}{2}\right) - \frac{1}{4} = \frac{5}{6}\left(\frac{17}{2} - 2\right)$$

$$\frac{65}{12} = \frac{65}{12}$$

$$\frac{17}{3} - \frac{1}{4} = \frac{85}{12} - \frac{10}{6}$$

2.

Solving Equations Manual

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II. Equations with Absolute Value:

★ Definition: The Absolute Value of a number is its distance from 0 on a number line.

Goal: Find the values of x that make the equation true.

- Steps:
- (1) First, get the absolute value by itself.
 - (2) Subtract (1) on one side, and add (1) on the other.
 - (3) Set the equation, that is inside the absolute value equal to 11 and -11
 - (4) Solve. and get x by itself.
 - (5) Add 3 to both sides of the equation
 - (6) Divide each side by 2!
 - (7) You end up with (2) solutions.

Solve Problem:

$$|2x - 3| + 1 = 12$$

-

$$\begin{array}{r} |2x - 3| - 1 = -12 \\ + 1 \quad + 1 \\ \hline 2x - 3 = -11 \\ + 3 \quad + 3 \\ \hline 2x = -8 \\ \frac{2x}{2} = \frac{-8}{2} \\ \hline x = -4 \end{array}$$

+

$$\begin{array}{r} |2x - 3| + 1 = 12 \\ - 1 \quad - 1 \\ \hline 2x - 3 = 11 \\ + 3 \quad + 3 \\ \hline 2x = 14 \\ \frac{2x}{2} = \frac{14}{2} \\ \hline x = 7 \end{array}$$

Check Problem:

($x = -4$)

$$2(-4) - 3 + 1 = -12$$

$$-8 - 3 + 1 = -12$$

$$-12 = -12$$

($x = 7$)

$$2(7) - 3 + 1 = 12$$

$$14 - 3 + 1 = 12$$

$$12 = 12$$

Both are solutions

3.

Solving Equations Manual

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III. Quadratic Equation

★ Definition: A Quadratic Equation is an equation containing the square and no higher power of the unknown quantity.

$$ax^2 + bx + c = 0 ; a \neq 0$$

Goal: Find the values of x that make the equation true.

3.) Factor method Steps:

- ① set the equation = 0
- ② Bring 18 over to the equation.
- ③ Factor the equation
- ④ Set each factor = to 0.

Solve Problem:

$$x^2 - 3x = 18$$

$$x^2 - 3x - 18 = 0$$

$$(x - 6)(x + 3) = 0$$

$$x - 6 = 0 \quad x + 3 = 0$$

$$\boxed{x = 6} \quad \boxed{x = -3}$$

Check Problem:

(f) $x = 6$

$$(6)^2 - 3(6) = 18$$

$$36 - 18 = 18$$

$$\boxed{18 = 18}$$

$$x^2 - 3x = 18$$

(f) $x = -3$

$$(-3)^2 - 3(-3) = 18$$

$$9 + 9 = 18$$

$$\boxed{18 = 18}$$

Both are solutions



④

Solving Equations Manual

④

III. Quadratic Equation (continue)

4.) Square Root Method:

★ Definition: A square root is a quantity of which a given quantity is the square.

Goal: Square both sides, and solve for (x) .

Steps:
① Take the square root of both sides of the equation.
② solve for (x)

Solve Problem:

$$(x-3)^2 = 12$$

$$\sqrt{(x-3)^2} = \sqrt{12}$$

$$x-3 = \sqrt{12} = \sqrt{4} \cdot \sqrt{3}$$

$$x-3 = 2\sqrt{3}$$

$$x = 3 \pm 2\sqrt{3}$$

$$x = 3 + 2\sqrt{3}$$

$$x = 6.46$$

$$x = 3 - 2\sqrt{3}$$

$$x = -0.46$$

✓ Check Problem:

$$f(x) = 6.46$$

$$(6.46 - 3)^2 = 12$$

$$41.73 + 9 = 12$$

32.

$$(x-3)^2 = 12$$

$$f(x) = -0.46$$

$$(-0.46 - 3)^2 = 12$$

$$(-2.116 + 9) = 12$$

Graph in calculator

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Solving Equations Manual

(Continue)

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III. Quadratic Equations

5) Quadratic Formula

★ Definition: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

- Steps:
- (1) move all constant terms right
 - (2) Get the leading coef. of (1)
 - (3) Complete the square
 - (4) Factor perfect square trinomial
 - (5) Use square root method

Goal: To get the solution of the quadratic formula.

Solve Problem:

$$3x^2 - 2x - 4 = 0$$

$$\frac{3x^2}{3} - \frac{2x}{3} = \frac{-4}{3}$$

$$x^2 - \frac{2}{3}x = -\frac{4}{3}$$

$$x^2 - \frac{2}{3}x = -\frac{4}{3} \quad -\frac{2}{3}\left(\frac{1}{2}\right) = \left(-\frac{2}{6}\right)^2 = \frac{4}{36} = \frac{1}{9}$$

$$x^2 - \frac{2}{3}x + \frac{1}{9} = -\frac{4}{3} + \frac{1}{9}$$

$$x^2 - \frac{2}{3}x + \frac{1}{9} = -\frac{11}{9}$$

$$\left(x - \frac{2}{6}\right)\left(x - \frac{2}{6}\right) = -\frac{11}{9}$$

$$\sqrt{\left(x - \frac{2}{6}\right)^2} = \sqrt{-\frac{13}{9}}$$

$$x - \frac{1}{3} = \pm \sqrt{\frac{13}{9}}$$

$$x = \frac{1}{3} \pm \sqrt{\frac{13}{9}}$$

$$x = \frac{1}{3} + \sqrt{\frac{13}{9}} \quad \text{or} \quad x = \frac{1}{3} - \sqrt{\frac{13}{9}}$$

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Solving Equations Manual

(Continue)

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III. Quadratic Equations:

6) Complete the Square

Steps:

- 1) Move the real numbers to the right
- 2) Take the coefficient of (x) and square it
- 3) Add the result to both sides of the equation.
- 4) Factor and take the square root
- 5) Solve for (x)
- 6) Move the real number to the right.

Solve Problem:

$$\begin{aligned}
 x^2 - 6x + 12 &= 0 \\
 x^2 - 6x + 9 &= -12 + 9 & \frac{1}{2}(-\frac{6}{1}) &= \frac{-6}{2} = \\
 x^2 - 6x + 9 &= -3 & & 3^2 = (9) \\
 (x-3)(x-3) &= -3 \\
 (x-3)^2 &= -3 \\
 \sqrt{(x-3)^2} &= \sqrt{-3} \\
 x-3 &= \sqrt{-3} \\
 +3 & \quad +3 \\
 x &= 3 \pm i\sqrt{3}
 \end{aligned}$$

$$x = 3 + i\sqrt{3} \quad \text{or} \quad x = 3 - i\sqrt{3}$$

Check:

Plug into Quadratic equation:

$$\left(\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \right)$$

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Solving Equations Manual

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IV. Cubic Equation:

★ Definition: An equation in which the highest power of the unknown quantity is a cube.

Goal: To find the values for (x) that make the equation.

Steps:

- (1) Subtract (1) from both sides.
- (2) Divide each side by (2).
- (3) Take the cube root ($\sqrt[3]{\dots}$) for both sides.
- (4) Get (x) by itself, from adding (4) from each side.
- (5) Check your results to find out which solution is the best.

Solve Problem:

$$2(x-4)^3 + 1 = 11$$

$$2(x-4)^3 + \underset{-1}{-1} = \underset{-1}{-1}$$

$$\frac{2(x-4)^3}{2} = \frac{10}{2}$$

$$(x-4)^3 = 5$$

$$\sqrt[3]{(x-4)^3} = \sqrt[3]{5}$$

$$x-4 = \sqrt[3]{5}$$

$$+x = \sqrt[3]{5}$$

$$x =$$

$$x = 4 \pm \sqrt[3]{5}$$

$$x = 4 + \sqrt[3]{5}$$

Check:

$$2(x-4)^3 + 1 = 11$$

$$2(4 + \sqrt[3]{5} - 4)^3 + 1 = 11$$

$$2(4 + 5 - 4)^3 + 1 = 11$$

$$2(5)^3 + 1 = 11$$

$$10 + 1 = 11$$

$$11 = 11$$

or

$$2(x-4)^3 + 1 = 11$$

$$2(4 - \sqrt[3]{5} - 4)^3 + 1 = 11$$

$$2(4 - 5 - 4)^3 + 1 = 11$$

$$2(-5)^3 + 1 = 11$$

$$\text{Not a solution} \quad -10 + 1 = 11$$

$$-9 = 11$$

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Solving Equations Manual

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IV. Rational Equations:

★ Definition: Equations that contain rational expressions.

Goal: To find all possible values for(x).

- Steps:
- ① First, find the (least common denominator)
 - ② Make sure to cancel the denominators.
 - ③ Add all like terms + distribute.
 - ④ Set everything = to (0)
 - ⑤ If it does not factor, we use the quadratic equation:

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve Problem:

$$\frac{x}{x+1} + \frac{2}{x} = 5$$

$$x \left(\frac{x+1}{1} \right) \frac{x}{x+1} + \frac{2}{x} * \left(\frac{x+1}{1} \right) = \frac{5}{1} x(x+1)$$

$$\begin{array}{r} x^2 + 2x + 2 \\ -x^2 \quad -2x \quad -2 \\ \hline \end{array} = 5x^2 + 5x$$

$$0 = 5x^2 + 5x - x^2 - 2x - 2$$

$$0 = 4x^2 + 3x - 2 \rightarrow \text{does not factor out}$$

Quadratic Formula:

$$\frac{-3 \pm \sqrt{(3)^2 - 4(4)(-2)}}{2(4)}$$

$$\frac{-3 \pm \sqrt{9 - 16(-2)}}{8}$$

$$\frac{-3 \pm \sqrt{9 - 32}}{8}$$

$$X = \frac{-3 \pm \sqrt{41}}{8}$$

✓ Check:

We graph the function:

$$\underbrace{\frac{x}{x+1}}_{y_1} + \underbrace{\frac{2}{x}}_{y_2} = \underbrace{5}_{y_2}$$

(calculator)

9.

Solving Equations Manual

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VI. Equations with Radicals:

★ Definition: A radical is or pertaining to
Or forming a root.

Goal: To find all the value or values
of (x).

- Steps:
- ① First, subtract 5 from each side
 - ② Then, multiply (2) on both sides
 - ③ Then square (x-5) by multiplying it by itself.
 - ④ Set the equation = 0
 - ⑤ Factor + solve for (x).

Solve Problem:

$$\begin{array}{r}
 X = \sqrt{X+7} + 5 \\
 -5 \qquad \qquad -5 \\
 \hline
 X-5 = \sqrt{X+7}
 \end{array}$$

$$(X-5)^2 = (\sqrt{X+7})^2$$

$$\begin{array}{l}
 (FOIL) \\
 (X-5)(X-5) = X+7
 \end{array}$$

$$X^2 - 5x - 5x + 25$$

$$X^2 - 10x + 25 = X + 7$$

$$X^2 - 11x + 18 = 0$$

$$(X-9)(X-2)$$

$$X-9 = 0$$

$$X-2 = 0$$

$X = 9$
 $X = 2 \rightarrow$ Not a solution

Check:

$$X = 9$$

$$9 \sqrt{9+7} + 5$$

$$9 = \sqrt{16} + 5$$

$$9 = 4 + 5$$

$$9 = 9$$

$$X = \sqrt{X+7} + 5$$

$$X = 2$$

$$2 = \sqrt{2+7} + 5$$

$$2 = \sqrt{9} + 5$$

$$2 = 3 + 5$$

$$2 \neq 8$$

VI. Equations with Radicals

10.) Cube Root

★ Definition: A cube root is a quantity of which a given quantity is the cube: $(\sqrt[3]{\quad})$.

Goal: Solve for (x)

- Steps:
- ① multiply each side by the power of (3).
 - ② Get (x) by itself
 - ③ Then, we add (1) to both sides.
 - ④ Divide (4) by both sides.
 - ⑤ Solve for (x)

Solve Problem:

$$\sqrt[3]{4x-1} = 2$$

$$(\sqrt[3]{4x-1})^3 = (2)^3$$

$$\begin{array}{r} 4x - 1 = 8 \\ +1 \quad +1 \end{array}$$

$$\frac{4x}{4} = \frac{9}{4}$$

$x = \frac{9}{4}$ is a solution.

Check:

$f(x) = 9/4$ $\sqrt[3]{4x-1} = 2$

$$\sqrt[3]{4(\frac{9}{4})-1} = 2$$

$$\sqrt[3]{\frac{36}{4}-1} = 2$$

$$\sqrt[3]{9-1} = 2$$

$$\sqrt[3]{8} = 2$$

$$\square 2 = 2$$

VII. Higher Order Polynomials. (degree > 3)

★ Definition: A polynomial is characterized by many or several names or terms.
 - To consist more than the degree of (3)

Goal: Factor the polynomials and solve for (x).

- Steps:
- (1) Use your calculator to graph the equation to find x-intercepts.
 - (2) Then set all (x-intercepts) to 0
 - (3) Find the degrees
 - (4) Factorize the equation.

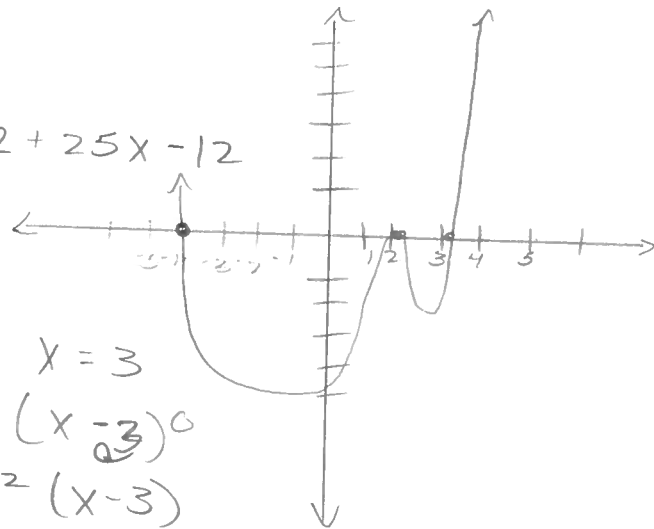
Solve Problem:

$$x^4 - x^3 - 13x^2 + 25x - 12 = 0$$

Graphed:

$$y_1 = x^4 - x^3 - 13x^2 + 25x - 12$$

$$y_2 = 0$$



$$x = -4 \quad x = 1 \quad x = 3$$

$$(x+4)^0 (x-1)^0 (x-3)^0$$

$$(x+4)(x-1)^2(x-3)$$

Check

Graphed

(Continue)

12.

Solving Equations Manual

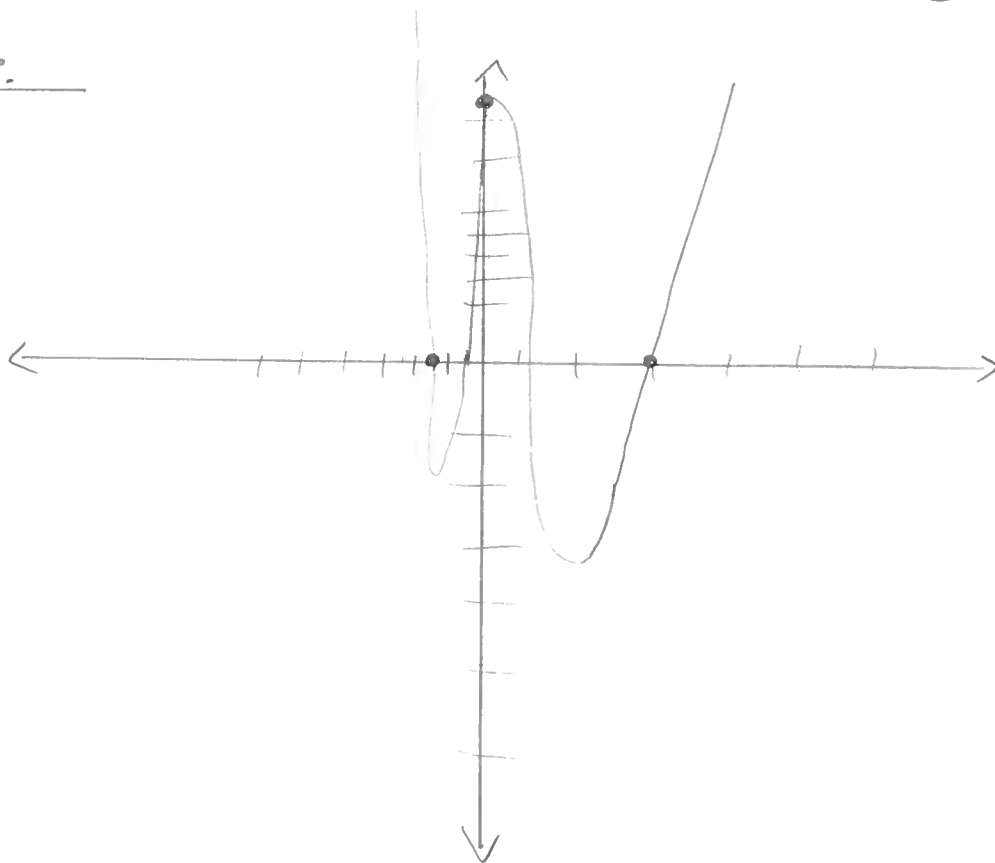
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VII. Higher Order Polynomials (degree > 3)

Solve Problem:

$$2x^4 - 5x^3 - 7x^2 + 10x + 6 = 0$$

Graphed:



$$2x^4 - 5x^3 - 7x^2 + 10x + 6 = 0$$

$$x = -1.414214$$

$$x = -1/2$$

$$x = 1.414214$$

$$x = 3$$

$$x = \pm\sqrt{2}$$

$$2(x - \sqrt{2})(x + \sqrt{2})(x + 1/2)(x - 3)$$

Check:

(Graphed in calculator).

VIII. Exponential Equations

★ Definition: having one or more unknown variables in one or more exponents.

Goal: To find the value of (x) Check and find the solutions.

- Steps: (1) First add "LOG" to both sides. (2) Have the exponent in front of the LOG (3) Get LOGS on one side (4) Divide + solve for x. (5) With (e^), get it by itself, by adding (6) Cancel lne + just have the exponent (7) Divide

Solve problem:

2^{x-4} = 25
Log 2^{x-4} = log 25
x-4 log 2 / log 2 = log 25 / log 2 -> plug in calculator

x-4 = 4.64
+4 +4
x = 8.64 -> is a solution.

Check

f(x) = 8.64
2^{x-4} = 25
2^{8.64-4} = 25
25 = 25

14.

Solving Equations Manual

(Continue)

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VIII. Exponential Equations:

14.) Solve Problem:

$$200 = 100e^{0.5x}$$

$$\frac{200}{100} = \frac{100e^{0.5x}}{100}$$

$$2 = e^{0.5x}$$

$$\ln 2 = \ln e^{0.5x}$$

$$\frac{\ln 2}{0.5} = \frac{0.5x}{0.5} \rightarrow \text{Plug in calculator.}$$

$$x = 1.386$$

Check:

$$200 = 100e^{0.5x}$$

$f(x) = 1.386 \rightarrow$ plug into equation

$$200 = 100e^{(0.5)(1.386)}$$

$$200 = (100)e^{0.693}$$

$$200 = 100(2)$$

$$\square 200 = 200$$

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Solving Equations Manual

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IX Logarithmic Equations

★ Definition: having a logarithmic as one or more.
Also, equations containing logarithmic expressions.

Goal: Find a solution for the value of (x) by using logarithmic strategies.

- Steps:
- (1) First, change the equation to an exponential equation.
 - (2) Get (x) by itself
 - (3) Find whether to add or subtract.
 - (4) Solve for (x)

Solve Problem:

$$\begin{aligned} \text{Log}(x-5) &= 2 \\ 10^2 &= x-5 \\ 100 &= x-5 \\ +5 & \quad +5 \\ \hline 105 &= x \end{aligned}$$

$$x = 105$$

✓ Check:

$$\begin{aligned} \text{Log}(x-5) &= 2 \\ f(x) = 105 &\rightarrow \text{plug into equation.} \end{aligned}$$

$$\begin{aligned} \text{Log}_{10}(105-5) &= 2 \\ \text{Log}_{10}(100) &= 2 \end{aligned}$$

$$\begin{aligned} 10^2 &= 100 \\ \checkmark 100 &= 100 \end{aligned}$$

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Solving Equations Manual

(Continue)

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IX. Logarithmic Equations:

16.) Solve Problem:

$$\ln(x+1) = 3$$

$$e^3 = x + 1$$

$$\begin{array}{r} e^3 = x + 1 \\ -1 \quad -1 \\ \hline \end{array}$$

$$e^3 - 1 = x \rightarrow \text{put in calculator.}$$

$$x = 19.086$$

Check:

$$\ln(x+1) = 3$$

$$f(x) = 19.086 \rightarrow \text{put in equation}$$

$$\ln(19.086 + 1) = 3$$

$$\ln(20.086) = 3$$

$$\checkmark 3 = 3$$