

Solving Equations Manual
MAC1105 – Fall 2006

Name: Key
 Class Days / Time: _____

I Linear Equations

___ 1) $\frac{2}{3}x - \frac{1}{4} = \frac{5}{6}(x - 2)$ $x = \frac{17}{2}$

II Equations with Absolute Value

___ 2) $|2x - 3| + 1 = 12$ $x = 7, -4$

III Quadratic Equations

___ 3) Factor Method: $x^2 - 3x = 18$ $x = 6, -3$
 ___ 4) Square Root Method: $(x - 3)^2 = 12$ $x = 3 \pm 2\sqrt{3}$
 ___ 5) Quadratic Formula: $3x^2 - 2x - 4 = 0$ $x = \frac{1 \pm \sqrt{13}}{3}$
 ___ 6) Complete the Square: $x^2 - 6x + 12 = 0$ $x = 3 \pm \sqrt{3}i$

IV Cubic Equation (Find One Real solution only)

___ 7) $2(x - 4)^3 + 1 = 11$ $x = 4 + \sqrt[3]{5}$

V Rational Equations

___ 8) $\frac{x}{x+1} + \frac{2}{x} = 5$ $x = \frac{-3 \pm \sqrt{41}}{8}$

VI Equations with Radicals

___ 9) Square Root: $x = \sqrt{x+7} + 5$ $x = 9$ ~~$x = 2$~~
 ___ 10) Cube Root: $\sqrt[3]{4x-1} = 2$ $x = \frac{9}{4}$

VII Higher Order Polynomials (degree > 3)

___ 11) $x^4 - x^3 - 13x^2 + 25x - 12 = 0$ $x = -4, 3, 1, 1$
 ___ 12) $2x^4 - 5x^3 - 7x^2 + 10x + 6 = 0$ $x = 3, -\frac{1}{2}, \pm\sqrt{2}$

VIII Exponential Equations

___ 13) $2^{x-4} = 25$ $x = \frac{\log 25}{\log 2} + 4 \approx 8.64$
 ___ 14) $200 = 100e^{0.5x}$ $x = \frac{\ln 2}{0.5} \approx 1.39$

IX Logarithmic Equations

___ 15) $\log(x - 5) = 2$ $x = 105$
 ___ 16) $\ln(x + 1) = 3$ $x = e^3 - 1 \approx 19.09$

Solving Equations Manual - Required Format

1. This Outline Sheet serves as your Cover Sheet and Table of Contents.
2. Start each equation on a new page, front side only. Number your equations.
3. **Outline a General Strategy** to solve the given type of equation.
4. **Solve each equation algebraically (by hand)**. Show your work solving the equation.
5. **Check your solution(s)** - either numerically or graphically.

MAC 1105 Solving Equations Manual

$$\textcircled{1} \quad \frac{2}{3}x - \frac{1}{4} = \frac{5}{6}(x-2)$$

$$\frac{\cancel{12}^4}{\cancel{7}^3} \frac{2}{\cancel{3}}x - \frac{1}{\cancel{4}^3} \frac{\cancel{3}}{\cancel{7}} = \frac{\cancel{12}^2}{\cancel{7}^1} \frac{5}{\cancel{6}}(x-2)$$

$$8x - 3 = 10(x-2)$$

$$8x - 3 = 10x - 20$$

$$-3 = 2x - 20$$

$$17 = 2x$$

$$x = \frac{17}{2}$$

$$\textcircled{2} \quad |2x-3| + 1 = 12$$

$$|2x-3| = 11$$

$$2x-3 = -11$$

$$2x = -8$$

$$x = -4$$

$$2x-3 = 11$$

$$2x = 14$$

$$x = 7$$

$$\textcircled{3} \quad x^2 - 3x = 18$$

$$x^2 - 3x - 18 = 0$$

$$(x+3)(x-6) = 0$$

$$x = -3 \quad x = 6$$

$$\textcircled{4} \quad (x-3)^2 = 12$$

$$\sqrt{(x-3)^2} = \pm\sqrt{12}$$

$$x-3 = \pm 2\sqrt{3}$$

$$x = 3 \pm 2\sqrt{3}$$

$$\textcircled{5} \quad 3x^2 - 2x - 4 = 0$$

$$a=3 \quad b=-2 \quad c=-4 \quad \leftarrow 4 - (-48)$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-4)}}{2(3)}$$

$$x = \frac{2 \pm \sqrt{52}}{6} = \frac{2 \pm 2\sqrt{13}}{6}$$

$$x = \frac{1 \pm \sqrt{13}}{3}$$

$$\textcircled{6} \quad x^2 - 6x + 12 = 0$$

$$x^2 - 6x + [9] = -12 + [9]$$

$$\frac{1}{2}(-6) \rightarrow -3, \quad (-3)^2 = 9$$

$$\sqrt{(x-3)^2} = \sqrt{-3}$$

$$x-3 = \pm \sqrt{3}i$$

$$x = 3 \pm \sqrt{3}i$$

$$\textcircled{7} \quad 2(x-4)^3 + 1 = 11$$

$$2(x-4)^3 = 10$$

$$\sqrt[3]{(x-4)^3} = \sqrt[3]{5}$$

$$x-4 = \sqrt[3]{5}$$

$$x = 4 + \sqrt[3]{5} \approx 5.71$$

$$\textcircled{8} \quad \frac{x}{x+1} + \frac{2}{x} = 5$$

$$\frac{\cancel{x}x(x+1)}{\cancel{x+1}} + \frac{2x(x+1)}{\cancel{x}} = \frac{5}{1}x(x+1)$$

$$x^2 + 2(x+1) = 5x(x+1)$$

$$x^2 + 2x + 2 = 5x^2 + 5x$$

$$2x + 2 = 4x^2 + 5x$$

$$0 = 4x^2 + 3x - 2$$

$$a=4 \quad b=3 \quad c=-2$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(4)(-2)}}{2(4)}$$

$$x = \frac{-3 \pm \sqrt{41}}{8}$$

$$\textcircled{9} \quad x = \sqrt{x+7} + 5$$

$$(x-5)^2 = (\sqrt{x+7})^2$$

$$x^2 - 10x + 25 = x + 7$$

$$x^2 - 11x + 18 = 0$$

$$(x-2)(x-9) = 0$$

$$\cancel{x=2} \quad x=9$$

$$\textcircled{10} \quad (\sqrt[3]{4x-1})^3 = (2)^3$$

$$4x-1 = 8$$

$$4x = 9$$

$$x = \frac{9}{4}$$

$$\textcircled{14} \quad x^5 - 2x^4 + 3x^3 - 6x^2 - 4x + 8 = 0$$

$$x = -1 \quad x = 1 \quad x = 2$$

$$\begin{array}{r} -1 \overline{) 1 \quad -2 \quad 3 \quad -6 \quad -4 \quad 8} \\ \quad \underline{-1 \quad 3 \quad -6 \quad 12 \quad -8} \\ 1 \overline{) 1 \quad -3 \quad 6 \quad -12 \quad 8 \quad \boxed{0}} \\ \quad \underline{1 \quad -2 \quad 4 \quad -8} \\ 2 \overline{) 1 \quad -2 \quad 4 \quad -8 \quad \boxed{0}} \\ \quad \underline{2 \quad 0 \quad 8} \\ 1 \quad 0 \quad 4 \quad \boxed{0} \\ x^2 + 0x + 4 \end{array}$$

$$x^2 + 4 = 0$$

$$\sqrt{x^2} = \sqrt{-4}$$

$$x = \pm 2i$$

$$x = -1 \quad x = 1 \quad x = 2 \quad x = \pm 2i$$

$$\textcircled{15} \quad \log_2^{x-4} = \log_2 25$$

$$(x-4) \log_2 = \log_2 25$$

$$x-4 = \frac{\log_2 25}{\log_2 2}$$

$$x = \frac{\log_2 25}{\log_2 2} + 4 \approx 8.644$$

$$\textcircled{16} \quad \frac{200}{100} = \frac{100}{100} e^{0.5x}$$

$$\ln 2 = \ln e^{0.5x}$$

$$\ln 2 = 0.5x$$

$$x = \frac{\ln 2}{0.5} \approx 1.386$$

$$\textcircled{17} \quad \log_2(x-5) = 3 \rightarrow 2^3 = x-5$$

$$8+5 = x$$

$$x = 13$$

$$\textcircled{18} \quad \ln(x+1) = 3$$

$$\log_{e^e}(x+1) = 3 \rightarrow e^3 = x+1$$

$$e^3 - 1 = x$$

$$x = e^3 - 1 \approx 19.086$$

$$\textcircled{19} \quad A = P(1+r)^t \quad \text{for } t$$

$$\log \frac{A}{P} = \log(1+r)^t$$

$$\log \frac{A}{P} = t \log(1+r)$$

$$t = \frac{\log \frac{A}{P}}{\log(1+r)}$$

$$\textcircled{20} \quad \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R}$$

$$R_2R + R_1R = R_1R_2$$

$$R(R_2 + R_1) = R_1R_2$$

$$R = \frac{R_1 R_2}{R_1 + R_2}$$